Null Steering in Partially Adaptive Arrays using Genetic Algorithms

by

Mohammed Jahangir Pasha

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

ELECTRICAL ENGINEERING

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This thesis, written by **Mohammed Jahangir Pasha** under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE** in **ELECTRICAL ENGINEERING**.

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عمادة الدراسات العليا
DEANSHIP OF GRADUATE STUDIES

Dedicated to

My

PARENTS,

SISTERS

and

BROTHERS

whose patience and perseverance

led me to this accomplishment.

Acknowledgements

In the name of Allah, Most Gracious, Most Merciful

"Read in the name of thy Lord and Cherisher, who created. Created man from a [leech-like] clot. Read and thy Lord is Most Bountiful. He Who taught [the use] of the pen. Taught man that which he knew not. Nay, but man doth transgress all bounds. In that he looketh upon himself as self-sufficient. Verily, to thy Lord is the return [of all]" (The Holy Quran, Surah 96)

All praise and glory be to Almighty Allah (SWT) who gave me courage and patience to carry out this work, and peace and blessings of Allah be upon Prophet Muhammad (PBUH). Acknowledge is due to King Fahd University of Petroleum and Minerals for providing support for this research.

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Abstract

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A new design technique, based on genetic algorithms, for null steering in Partially adaptive antenna arrays by controlling the minimum number of element positions is presented. This technique shows the possibility of placing nulls in the radiation pattern to suppress interference and unwanted signals, with minimum cost and complexity. In this work, a new approach to minimizing the number of controlled elements and sidelobe variation is developed using genetic algorithms. The developed procedure selects those degrees of freedom which minimizes both the number of controlled elements and sidelobe variation simultaneously while steering nulls in the desired directions. Uniform and Chebyshev partially adaptive arrays of various sizes are studied for single and multiple nulls. The results show that the number of controlled elements can be reduced to less than half, which leads to a significant reduction in the number of stepper motors used as compared to the fully adaptive implementation. Consequently, this technique minimizes the complexity, cost and at the same time maintains almost the same performance as that of fully adaptive arrays.

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خلاصة الرسالة

إسم الطـــالب : محمد جهانغير باشاه

عنوان الرسالة : توجية إنعدامات الإشعاع في المصفوفات الهوائية المتكيفة جزئيا بإستعمال الخوارزمات الجينية.

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لقد تم تصميم تقنية جديدة , بإستعمال الخوارزمات الجينية لتوجيه إنعدامية الإشعاع في المصفوفات الهوائيةالمتكيفة جزئيثا عن طريق التحكم في مواضع عدد أدنـــى من وحدات المصفوفات . أظهرة التقنية المقترحة إمكانية وضع إنعدامية الإشعاع داخل نمط إشـــعاع المصفوفات لإلغاء إشارات التداخل والتشويش الغير المرغوب فيها , بتكلفة أقلْ وتصميم أسهل . طور هذا البحث طريقة جديدة لإختزال عدد وحدات المصفوفات المتحكم فيها وتغير نصف الإشعاع الثانوي . تختار الطريقة المقترحة درجات الحرية المتميزة بـــإختزال عدد الوحدات المتحكم فيها وتغير الإشعاع الثانوي وفي نفس الوقت يتم توجيه إنعدامية الإشعاع إلى أي إتجاة. قام البحث بدراسة مصفوفات هوائية متكيفة جزئيا من نوعـــي منتظم Chebyshev باحجام مختلفة في حالة إنعدامية إشعاع واحد ومتعدد . لقد برهنـــت النتائج أنة يمكن إختزال عدد الوحدات ألى أقل من النصف , مما يؤدي ألى تقليل كبـــير في عدد محركات التوجيه مقارنة بالمصفوفات الهوائية المتكيفة كليـــا . وبهـــذا مكنـــت الطريقة المقترحة في تسهيل تصميم المصفوفات الهوائية وترشيد تكلفتها مع المحافظة علـــى نفس أداء المصفوفات المتكيفة كليا .

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Chapter 1

INTRODUCTION

1.1 Motivation

Adaptive antenna arrays are extensively used in applications such as radar, sonar and communication systems. Adaptive array processing techniques are currently the subject of extensive investigation. The main reason for this investigation is that the interference signals can be suppressed by adaptively steering the nulls to reduce the sidelobe levels in the direction of interference and unwanted signals while keeping the main beam pointing towards the desired signal. This interference suppression capability is a principle advantage of adaptive arrays compared to conventional waveform processing techniques.

Synthesis techniques which provide control of the nulls of the antenna array pattern are of great interest. They represent generalizations of classical pattern synthesis techniques, where the main beam shape and sidelobe level are more important than the detailed sidelobe structure. In general, these methods involve the search for an appropriate set of complex weights (full amplitude/phase), phase-only or amplitude-only variations, element positions along (or perpendicular to) array element axis, so that the main beam remains pointing towards the desired signal, while nulls are formed in the directions of strong undesired sources.

In full adaptive arrays using element position perturbation method, positions of each element is adaptively controlled by using a high precision stepper motor at each element. But for large arrays controlling every element individually can become very expensive, unrealistic and difficult with large number of motors and also it takes a lot of time to form the nulls [1]. Therefore it is highly desirable to reduce the number of perturbed elements (motors) while maintaining the array pattern approximately the same as in the case of full adaptive array. Hence, in partially adaptive arrays only 'K' elements out of 'N' elements are perturbed with Δ_K where $K < N$.

Most of the work concerned with partially adaptive nulling has been directed towards maximizing a performance index such as signal-to-noise and interference ratio (SNIR) [2][3][4]. Also the techniques based on eigenvalues and eigenvectors are becoming more popular. Most investigators are using the eigenstructure of the signal and/or noise correlation matrices to design partially adaptive arrays [5][6][7]. On the other hand, design of partially adaptive arrays using synthesis techniques is rarely found in the literature. The aim of this work is to steer the nulls in partially adaptive arrays based on synthesis technique, employing the horizontal type of element position perturbation method using genetic algorithms.

The *array pattern nulling* and *controlled elements minimization* are the two important phases in the design of partially adaptive arrays. While *array pattern nulling* steers the nulls in the desired interference directions, *controlled elements minimization* simultaneously minimizes the number of controlled elements required for array pattern nulling. Therefore *null Steering* and *minimization of the number of controlled elements* are closely interrelated, and are dealt with simultaneously.

The objective of null steering is to steer the nulls in the given interference directions with out affecting the main beam and the objective of minimizing the number of controlled elements (partial adaptivity) is to minimize the amount of hardware cost which is mainly the cost of high precision stepper motors and to decrease the complexity. The *objective function* involved in the optimization process for the design of partially adaptive arrays is complex. Several optimization techniques can be used for this purpose such as *Simulated annealing* [8] and *tabu search* [9]. **Genetic algorithm** (GA) is another promising global optimization technique [10]. It works by emulating the natural process of evolution as a means of progressing towards the optimum.

In the early 1970's, John Holland incorporated the features of natural evolution to yield a technique for solving difficult problems which he later named genetic algorithm. Genetic algorithm (GA) provides robust search in complex spaces. These algorithms are computationally simple and can be easily improved. They are not limited by restrictive assumptions about the search space. They efficiently exploit historical information to speculate on the new search with an expected improved performance. Those were the major advantages of the genetic algorithm technique. The algorithm starts with a *population* which consists of several solutions to the optimization problem. A member of population is called an *individual.* A *fitness* value is associated with each individual. Each solution in the population or an individual is encoded as a string of symbols. These symbols are known as *genes* and the solution string is called the *chromosome.* The values taken by genes are called *alleles.* The Population at a given stage is referred to as a generation. Several pair of individuals (parents) in the population *mate* to produce offsprings by applying the genetic operator *crossover.* Each offspring undergoes *mutation* with probability called *mutation rate.* Selection of parents is done by repeated use of *parent selection* function. A number of individuals and offsprings are passed to a new *generation* such that the number of individuals in the new population is the same as old population. A *selection* function determines which strings form the population in the next generation.

In evolution, the problem each individual faces is one of searching for beneficial adaptations to a complicated and changing environment. The *knowledge* that each individual has gained is embodied in the makeup of its chromosome. The operations that alter this chromosomal makeup are applied when parents reproduce. Random mutation provides background variation and occasionally introduces beneficial material into an individual's chromosome. Crossover exchanges corresponding genetic material from two parent chromosomes, allowing beneficial genes on different parents to be combined in their offspring [11].

The genetic algorithms differs from the other stochastic techniques by being able to encode and exploit past information efficiently during a search. This learning capability provides the genetic algorithm with a guiding capability for searching efficiently through a complex multi-dimensional search space. The key steps in the application of genetic algorithm include an appropriate string encoding or chromosomal representation, a way to create an initial population of solutions, and an effective objective function.

The basic difference between genetic algorithms and classical search methods are:

- GA's work with a coding of the parameters set, not the parameters themselves.
- GA's search from a population of points, not a single point.
- GA's use payoff (objective function) information, not derivatives or other auxiliary knowledge.
- GA's use probabilisetic rules, not deterministic rules.

1.2 Proposed Objectives

- To steer a null and multiple nulls in partially adaptive arrays based on element position perturbation technique using genetic algorithms.
- To develop an algorithm that determines the minimum number of elements and their locations in partially adaptive arrays which can be controlled to obtain the required performance as that of the full adaptive array using element position perturbations technique. The objective function will be based on forming the nulls in the required direction and in the mean time it wiii include the optimum(minimum) number of controlled elements.
- To present theoretical study as well as simulation results of the genetic algorithm technique and its application to partially adaptive arrays with varying number of controlled array elements.
- To study the effects of varying the number of nulls on the array performance.
- To study the effect of reducing the number of controlled elements on the array parameters such as the half-power beamwidth(HPBW) and array directivity.

Chapter 2

Literature Review

2.1 Synthesis Techniques

Synthesis techniques which provide control of the nulls of the array pattern are of great interest for two reasons. First, they represent generalizations of classical pattern synthesis techniques, which deal with the main beam shape and sidelobe level, that are more important than the detailed sidelobe structure. The second reason is that they provide insight into adaptive antenna systems. Several methods for the synthesis of array antenna pattern with prescribed nulls exist in the literature and will be discussed briefly as follows.

2.1.1 Full amplitude/phase control method

This is the most efficient method and is based on full amplitude and phase control at each array element [12], but it is very costly choice. The method, for pattern null synthesis, starts from a given original pattern, with desired main beam and sidelobe envelope, corresponding to a given element coefficients a_n and an initial equal spacing between elements d_0. Both amplitude and phase of these element coefficients are then perturbed such that the perturbed pattern has nulls at the desired directions, where their locations are assumed to be known.

2.1.2 Phase only control method

In this method only phase of current at each array element is controlled. This method starts from a given original pattern with desired main beam, sidelobes & nulls with element coefficients a_n and equal spacing between elements d_0 then only phase of these coefficients are perturbed such that the pattern has nulls at desired directions [13].

Several publications have appeared in the literature regarding this method [13][14][15]. In particular, the case with small phase perturbations are distinguished from that of arbitrary large phase values. That can be achieved by placing a relatively small number of nulls in the region of low sidelobes, which constitutes a relatively modest pattern perturbation. However, a general limitation of this approach is that it is incapable of realizing two nulls which are imposed perfectly symmetrically about the main beam.

Moreover, large phase perturbations are required when a null is imposed in the main beam vicinity, when multiple nulls are imposed within a relatively narrow angular sector, or when the number of nulls become large relatively to the array element number. Large phase perturbations null synthesis problem seems amenable only to numerical solution using the nonlinear programming algorithms [16]. The symmetry of the original pattern will be lost in the perturbed pattern. This approach solves the problem at the cost of main beam gain and substantial increase in sidelobe level over the rest of the pattern.

2.1.3 Amplitude only control method

Here, null steering can be achieved by controlling the current amplitude only which overcomes some of the limitations of the phase only method while simplifying the adaptive system. So, in this case phase shifter's are used solely for steering the main beam towards the desired signal.

Vu [17] described a method of null steering without using phase shifter's. This is done by forcing the zeros of the array factor to occur in conjugate pairs on the unit circle in the complex plane. The paper also showed that if the number of jammer's is much smaller than half the total number of elements in the array, it is possible to optimize the pattern as well as suppress the jammer's. In a classic paper Schelkunoff

[18] presented a method that is conducive to the synthesis of arrays whose patterns possess nulls in the desired directions. This method requires information of the number of nulls and their locations. The number of elements and their excitation coefficients are then derived.

2.1.4 Element position perturbations method

This technique was developed and verified by Dawoud and Ismail [19] [20], where the process of null steering is carried out by controlling the positions of the antenna array elements. The experimental results proved the validity of null steering by controlling the element positions.

In general, there are two methods to achieve null steering in this technique. It starts from a given original pattern with desired main beam, sidelobe and nulls with element coefficients 'a_n' and equal spacing between elements 'd_0' then in the first method the positions of the antenna array elements are perturbed along the array elements axis (horizontally) [19] such that the pattern has nulls at desired directions. In the second method the positions of the antenna elements are perturbed perpendicular to the array element axis (vertically) [21] towards the broadside direction such that the pattern has nulls at desired directions. A brief discussion of the first method which is applied in the present work is given in the next chapter.

M. M. Dawoud [22] presented a methodology of null steering in scanned arrays by element position perturbations. This method was devised to improve the performance of scanned arrays based on the null steering criterion, keeping beamwidth and sidelobe variations to a minimum.

2.2 Partially adaptive arrays

Partially adaptive array processing techniques are currently the subject of extensive investigation and have found applications in radar, sonar and communication systems due to their effectiveness in cancelling strong interference signals and at the same time reducing the complexity and cost. A partially adaptive array is one in which elements are controlled in groups (the subarray approach) or in which only certain elements called auxiliary elements are made controllable.

A fully adaptive array in which every element is individually controlled is obviously preferred as it affords the greatest control over the array response. For arrays containing large number of elements, individually controlling every element can prove a prohibitive task. In addition, reducing adaptive dimensions can result in a faster adaptive response.

Partially adaptive array concept have been the subject of many investigations [2] [3] [23] [4] [5] [6] [7]. This area has started to be exploited by the middle of 1970's. Chapman [2], has examined several partially adaptive approaches for distributed interference cancellation. The effect of the random sidelobe level caused by element weighting errors was shown to adversely affect the performance inversely proportional to the number of adaptive elements employed. This is somewhat expected since the distributed interference model calls for a number of degrees of freedom approaching that of the full array.

Morgan [3] studied the multiple sidelobe canceller (MSC). The MSC adaptively combines the outputs of a few array elements called auxillaries and subtracts it from a fixed main beam formed using all the elements. Morgan employed extensive simulations to determine rules of thumb for auxiliary element selection when the array is linear, equispaced and operate in a narrow-band environment.

Ismail El-Azhary et al [23] proposed the concept of partial adaptivity using only edge elements. They showed that the far-field lobes of the edge elements of a uniformly excited linear array are nearly equal in width to the sidelobes of the array and hence the edge elements can be used for cancellation of specific sidelobes of the pattern. But this approach does not lead to determine the minimum number of elements that can be controlled in partially adaptive arrays. Van Veen and Roberts [4] proposed the use of a fixed transformation that maps the fully adaptive space into a partially adaptive space based on minimization of output interference power. Techniques based on eigenvalues and eigen vectors are becoming more popular. Most investigators are using the eigen structure of the signal and /or noise correlation matrices to design partially adaptive arrays [5] [6] [7].

Van Veen , Barry D. [5] showed that for arbitrary linearly constrained minimum variance beamformers the required adaptive dimension is less than or equal to the rank of the spatially / temporally correlated portion of the interference correlation matrix and knowledge of the eigenstructure of the interference correlation mătrix is required to implement a beamformer with this adaptive dimension. To avoid adaptive estimation of the eigenstructure, the eigenstructure of an averaged correlation matrix is utilized and the adaptive dimension is given by the rank of the averaged correlation matrix.

In a recent paper [6], ER M.H. et al proposed a new approach to the design of a uniform linear array with partial adaptivity. The technique uses the symmetric and skew-symmetric eigen vectors of a certain toeplitz matrix to form a fixed weighting vector for the main beam and a signal blocking matrix for the cancellation beams. This approach allows the adaptive dimensions of the processor to be reduced to about half of that of the full processor without prior knowledge of the interference field.

In another recent paper [7] Y.H. Sng et al proposed an alternative approach to designing a partially adaptive array using direction-of-arrival (DOA) estimation and a null steering technique, in which DOAs of the interference are estimated using some selected outputs of the signal blocking matrix to form the ensemble signal blocking correlation matrix. The eigen vector corresponding to the minimum eigen value of the ensemble signal blocking correlation matrix is then used to form nulls in the spatial spectrum which are corresponding to the incident angles of interferences. Synthesized nulls are then placed in the main beam to cancel all the interference signals. This approach reduced the adaptive dimensions of the processor to less than half of that of the full processor. But this technique does not lead to identifying which elements are being controlled.

Hu et al [24] in a recent publication proposed a synthesis method to design an array pattern with a deep null by controlling only the excitation phases of part elements by using a weight function to construct an objective function which was then minimized.

R.T. Al-Mushcab [25] conducted the study of the design of partially adaptive arrays using element position perturbation to investigate the effect of reducing the number of controlled elements on the array performance. The partially adaptive method presented in his thesis provides the element position perturbations for q selected elements ($1 \leq q \leq N$) where N is the number of array elements. But it does not provide the minimum number of elements to be selected or their locations.

From the above discussion, it is found that, No definite solution for selecting the number and position of the controlled elements has been reported. On the other hand, design of partially adaptive array using synthesis technique is rarely found in the literature.

2.3 Genetic Algorithm Optimization and its application to Antenna Design

Conventional functional optimization techniques are based on greedy, local optimization methods such as gradient methods. An example of such methods is the random walk solution space searches. These conventional techniques are often poorly suited to the task of arbitrary pattern synthesis in antenna arrays due to high dimensional, multiobjective functional domains involved. In addition, traditional optimization techniques usually require the objective function to be, at the least, continuous and in many cases to be differentiable, placing severe limitations on the form and content of the objective function. This work presents a different and relatively new functional optimization methodology known as Genetic algorithm (GA) optimization that overcomes the above mentioned problems of the traditional techniques. Genetic algorithms have been successfully applied to a number of Antenna design problems.

M. M. Dawoud et al [26] applied the genetic algorithm to solve the problem of null steering in adaptive arrays. They showed that using this technique it is possible to steer nulls precisely to the required interference directions and achieve any prescribed null depth. The paper also showed the potential of the GA for conformal array design.

M. M. Dawoud et al [27] made a comparison of the results obtained from analytic solutions for element position control, with the GA approach and showed some of the advantages accrued by using the GA for null steering in linear array patterns.

B Chambers et al [28] presented results using genetic algorithm for the case of an end-fire array antenna with both adaptive nulling and radiation pattern envelope limitations applied and for the case of wideband, high performance cylindrical radar absorbers.

A. Alphones and V. passoupathi [29], has proposed that the use of genetic algorithm removes the restrictions on the array element displacements, required in the case of linear analytic method and also showed the results obtained for axial, elevational and arbitrary directional element position perturbations through genetic algorithm for imposing nulls in the desired directions in radiation pattern.

Randy C Haupt [30], has compared the application of genetic algorithms and traditional gradient-based algorithms to various optimization problems in electromagnetics and showed that gradient algorithms work well for a small number of continuous parameters while genetic algorithms are best for a large number of quantized parameters.

D. Marcano et al [31] has presented the synthesis of the radiation pattern of a linear array subjected to many restrictions and the synthesis of dual beam radiation pattern with low sidelobes using a genetic algorithm.

Randy L. Haupt [32], has proposed how to optimally thin an array using genetic algorithms. The genetic algorithm developed determines which elements are turned off in a periodic array to yield the lowest maximum relative sidelobe level. Simulation results for a 200 element linear and planar arrays are shown. The arrays are thinned to obtain a sidelobe level of less than -20 dB.

D. Marcano et al [33] has presented two methods for the synthesis of the radiation pattern of a linear array antenna based on genetic algorithms, with these methods it is possible to synthesize a radiation pattern specified by shaped beams and nulls in the given directions.

A recent paper by Johnson et al [34] discusses the use of genetic algorithms for a variety of problems in electromagnetics such as design of shaped beam antenna arrays. the design of broadband patch antennas etc. It concludes by emphasizing the suitability of the genetic algorithm for a broad class of electromagnetic problems.

Marcano et al [35] applied the genetic algorithm for the synthesis of linear antenna arrays and concluded that the synthesis of the radiation pattern with many constraints is a non linear optimization problem, which is difficult to solve using methods based on deterministic rules because of local minima problems and hence, they suggested the genetic algorithm as a solution.

Haupt [36] tried to overcome the large time consumption problem, when arrays are numerically optimized by a new method that ensures a fast convergence of the genetic algorithm. This is possible if the array parameters are encoded with a gray code and therefore, are less likely to be disturbed during the crossover operation.

Johnson et al [37] discussed a number of applications of genetic algorithm such as design of light weight, broadband microwave absorbers, the reduction of array sidelobes in thinned arrays, the design of shaped beam antenna arrays etc. They concluded that the GA optimization is suitable for a broad class of problems related to aerospace antennas and electromagnetics.

In another recent publication Ares et al [38] used both simulated annealing and genetic algorithms to find optimum excitation for patterns with null filling. These methods have the advantage that the optimum aperture distribution is found without searching the entire solution space. A comparison between the performance of both methods shows that the GA's are faster than simulated annealing for this problem. This partial list is a good indicator of the applicability of genetic algorithm to diverse antenna problems.

Chapter 3

Partially Adaptive Arrays

3.1 Element position perturbations

In this technique, the nulls are formed in the desired directions by means of perturbations to the element positions. This approach is based on direct synthesis of array pattern with the array nulls in the desired directions where both amplitudes and phases of the original array are kept unchanged, in contrast to the usual iteration solution resulting from using adaptive algorithms.

In this section the horizontal position perturbation technique is presented. In this work, we are not concerned with the various methods of locating undesired sources, they are assumed to be known.

A linear array of N isotropic equispaced elements is considered, The corresponding

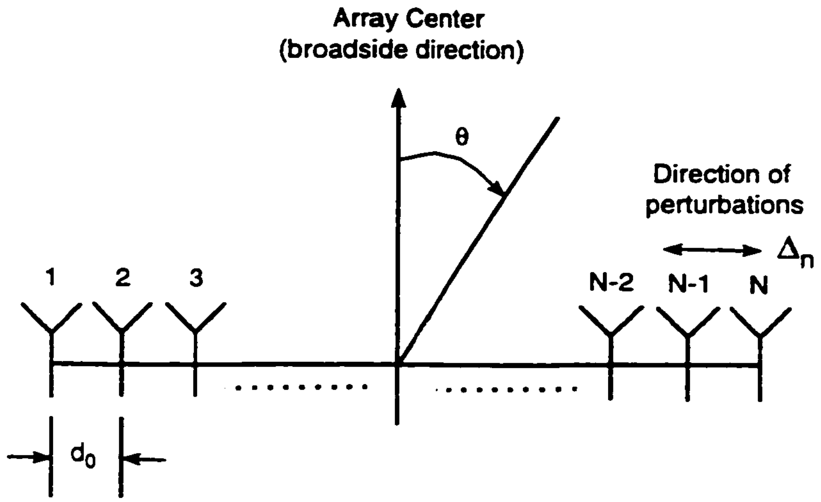


Figure 3.1: Linear array elements with perturbations along the array element axis

geometry is shown in Figure 3.1 which has the Array Factor [39].

$$F(u) = \sum_{n=1}^{N} a_n e^{jd_n(u-u_s)} \tag{3.1}$$

Where :

$u = kSin(\theta)$, θ is angle from broadside.

$u_s = kSin(\theta_s)$, θ_s is the steering angle of the main beam from broadside

a_n=element excitation coefficients.

d_n=d_0[n-N/2-0.5]

d_0=the interelement spacing in wavelength λ

$k = \frac{2\pi}{\lambda}$(wave number), λ is the incident or radiated wavelength

The following analysis has been derived in [39] and summarized here for reference. Because the element positions reference is taken to be the center of the array, the element positions d_n have odd symmetry with respect to the array position reference center that is,

$$d_n = -d_{N-n+1} \tag{3.2}$$

Where n = 1,2.......,N

This method starts from a given original pattern $F_0(u)$, with desired main beam and sidelobe envelope, corresponding to a given element coefficients a_n and an initial interelement spacing equal d_0. These elements are then perturbed horizontally such that the perturbed pattern has nulls at the desired directions, where their locations are assumed to be known.

Assume a position change of Δ_n will produce the required nulls in the far field pattern. The new element positions are represented by

$$x_n = d_n + \Delta_n \tag{3.3}$$

when there is no position change, $\Delta_n = 0$, then $x_n = d_n$. The perturbed pattern corresponding to the perturbed element positions, x_n, can be written as,

$$F(u) = \sum_{n=1}^{N} a_n e^{jx_n(u-u_s)} \tag{3.4}$$

$$= \sum_{n=1}^{N} a_n e^{j\Delta_n(u-u_s)} e^{jd_n(u-u_s)} \tag{3.5}$$

The above result can be written as,

$$F(u) = \sum_{n=1}^{N} W_n e^{jd_n(u-u_s)} \tag{3.6}$$

where, $W_n = a_n e^{j\Delta_n(u-u_s)}$

Equation 3.6 shows that the perturbed pattern can be expressed as if the original current coefficients 'a_n' are perturbed to have the values 'W_n', which places nulls in the prescribed directions.

In general, M nulls are required in the pattern to cancel M jammers at angular locations u_m. Now we wish to determine the perturbations Δ_n to place the nulls at the M locations u_m, or equivalently find the solution to the equation given below.

$$F(u_m) = \sum_{n=1}^{N} a_n e^{j\Delta_n(u_m-u_s)} e^{jd_n(u_m-u_s)} = 0 \tag{3.7}$$

$$= \sum_{n=1}^{N} a_n e^{j(d_n+\Delta_n)(u_m-u_s)} = 0 \tag{3.8}$$

Where $u_m = kSin(\theta_m)$, θ_m is the angle where nulls are required. m = 1,2,....M

The problem of finding the element position perturbations $\Delta_n's$ is a nonlinear problem which does not have a linear solution. Therefore, the null synthesis method will be based on small element position perturbations $\Delta_n \ll d_0$ (interelement spacing) which can only be linearized by using two term taylor expansion of the element position perturbation phase term, which results in analytic solution [39]. This approach was achieved by placing relatively small number of nulls ($M \ll N$) in the sidelobe region.

The required element position perturbations can be calculated analytically according to [39] as

$$\Delta_n = \sum_{m=1}^{M} C_m a_n \sin[d_n(u_m - u_s)] \tag{3.9}$$

Where M is the number of steered nulls and C_m represents the beam coefficient of the m^{th} cancellation beam.

The nonlinear equation 3.8 can be solved using genetic algorithms [40]. The use of genetic algorithm removes the restrictions on the array element displacement, required in the case of linear analytic method. The element position perturbations $\Delta_n' s$ are selected as the input variables to the genetic algorithm solution. This can be expressed as a chromosome vector V of length N, where N is the number of array elements.

$$V = [\Delta_1, \Delta_2, ..., \Delta_N]$$

Where Δ_n is the position perturbation of element n, $n = 1, 2, ..., N$

The main advantages of null steering using element position perturbation are

- Amplitude and phases are solely used to control the shape of the pattern and also to steer the main beam to any required direction. Thus if the interference direction is known exactly then the complete nulling of the noise source can be achieved despite the coarseness of the phase increments.

- A null located in certain direction will produce an image null about the center of the array. So, the computational time can be effectively halved.
- The feed network for such arrays will be greatly simplified. Therefore null steering using element position perturbation is less complicated compared with the other techniques [39].

3.2 Partially Adaptive Arrays based on element position perturbation method

Adaptive antenna arrays are widely used in many advanced radar, sonar and communication systems because of their effectiveness in cancelling strong interferences that are received on a sidelobe of the main antenna pattern [16]. Figure 3.2 shows

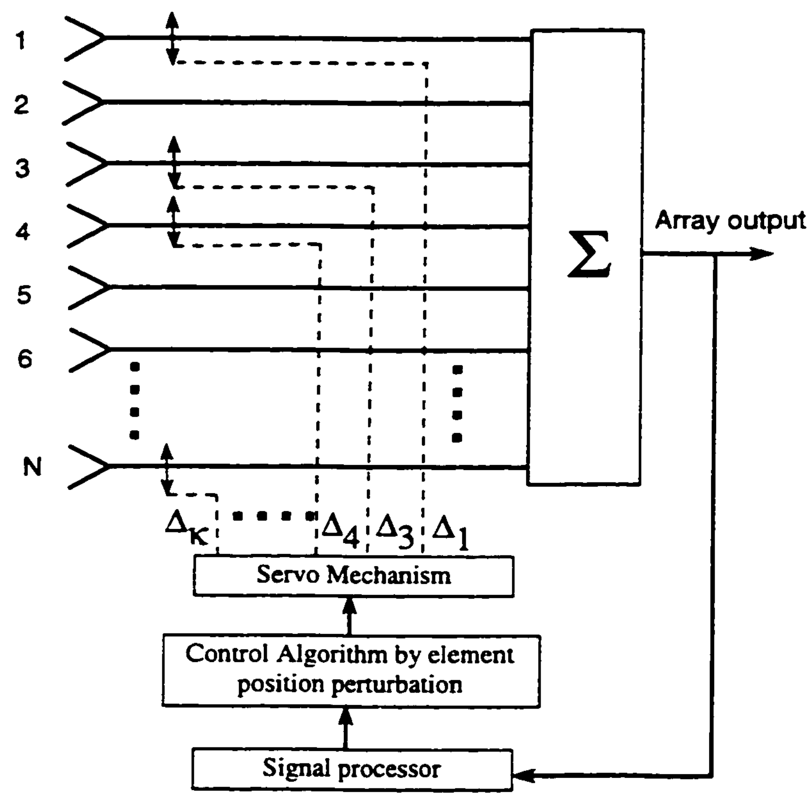


Figure 3.2: Generic block diagram of adaptive array by element position perturbations controlling minimum number of elements K

a generic block diagram of the element position perturbation adaptive array, as described in [25]. Here a main array is formed by combining N elements in a conventional manner. A computer or a microprocessor with a real time adaptive control algorithm controls the position perturbations Δ_K of K elements using high precision stepper motors. If $K = N$, then the array is said to be fully adaptive. In case of $K < N$, the array is said to be partially adaptive which is the subject of the study. The fully adaptive array in which every element is individually adaptively controlled is obviously preferred since it affords the greatest control over the array response [41]. But, for large arrays, individually controlling every element can prove a prohibitive implementation expense. Further more, the arrays using element position perturbation technique become more difficult, costly and unrealistic to jam the undesired signals with large number of motors. Consequently it is highly desirable to reduce the number of perturbed elements while maintaining a high degree of control over the array response. This can be done by adapting one of the following adaptive control philosophies [42].

1. Select only a fraction of the array elements to be adaptively perturbed. This will result in an elemental level adaptivity.

2. Group the N elements in the array into M subarrays, and adaptively control each of the resulting subarray.

For the nulls imposed in the sidelobe region, minimum selected elements in the aperture is the preferred approach [16]. For main beam nulling, this approach will not be efficient since the elements would have to be driven with very large amplitudes. In this case subarray nulling is preferable since each subarray provides a large beam in the main beam direction and therefore these beams can be used efficiently to create a main beam null. Since in the present work nulls are steered in the sidelobe region, the partially adaptive array with minimum selected number of elements is the preferred method.

In this section, the partially adaptive linear arrays employing the horizontal type of element position perturbations technique is described. The following analysis has been derived in [25] and summarized here for reference.

The computation of the perturbations , Δ_q, for a pre-selected number of q elements, starts by considering a linear array of N isotropic equispaced elements, which has an array factor

$$F(u) = \sum_{n=1}^{N} a_n e^{jd_n(u-u_s)} \tag{3.10}$$

Where :

$u = kSin(\theta)$, θ is angle from broadside.

$u_s = kSin(\theta_s)$, θ_s is the steering angle of the main beam from broadside.

a_n=element excitation coefficients.

d_n=d_0[n-N/2-0.5]

d_0=the interelement spacing in wavelength λ

$k = \frac{2\pi}{\lambda}$(wave number), λ is the incident or radiated wavelength.

Assume the change of position $\hat{\Delta}_n$ produces the required nulls for the far field pattern.

$$\hat{\Delta}_n = 0 \qquad if \qquad n \notin q \tag{3.11}$$

$$= \Delta_q \qquad if \qquad n \in q \tag{3.12}$$

where the pre-selected elements are denoted by q, which is a subset of the array elements N. Then the new element positions are given by

$$x_n = d_n + \hat{\Delta}_n \tag{3.13}$$

when there is no element position change, $\hat{\Delta}_n = 0$ and $x_n = d_n$. The perturbed pattern corresponding to the perturbed element positions, x_n, can be written as

$$F(u) = \sum_{n=1}^{N} a_n e^{jx_n(u-u_s)} \tag{3.14}$$

$$= \sum_{n=1}^{N} a_n e^{jd_n(u-u_s)} e^{j\hat{\Delta}_n(u-u_s)} \tag{3.15}$$

In general, M nulls are required in the pattern to cancel M jammers at angular location u_m $(1 \leq m \leq M)$. we now wish to determine the perturbations $\hat{\Delta}_n$ to place nulls at the M locations u_m or equivalently find the solution to the equation given below.

$$F(u_m) = \sum_{n=1}^{N} a_n e^{jd_n(u_m - u_s)} e^{j\hat{\Delta}_n(u_m - u_s)} = 0 \tag{3.16}$$

The problem of finding the minimum element position perturbations $\hat{\Delta}_n$ is a non-linear problem, which does not have a linear solution. Therefore, the null synthesis method will be based on small element position perturbations $\hat{\Delta}_n \ll d_0$ (interelement spacing) which can only be linearized by using two term taylor expansion of the element position perturbation phase term, which results in analytic solution [25]. This approach was achieved by placing relatively small number of nulls $(M \ll N)$ in the sidelobe region.

The required element position perturbations for a pre-selected number of q elements can be calculated analytically according to [25] as

$$\Delta_q = \sum_{m=1}^{M} C_m a_q \sin[d_q(u_m - u_s)] \tag{3.17}$$

Where M is the number of steered nulls and C_m represents the beam coefficient of the m^{th} cancellation beam.

The partially adaptive method presented in reference [25] only provides the element position perturbations for q selected elements $(1 \leq q \leq N)$. But it does not provide the minimum number of elements to be selected or their locations.

The problem of finding the minimum no. of elements and their locations can be obtained from solving

$$F(u_m) = \sum_{n=1}^{N} a_n e^{jd_n(u_m - u_s)} e^{j\hat{\Delta}_n(u_m - u_s)} = 0 \tag{3.18}$$

Where

$$\hat{\Delta}_n = 0 \qquad if \qquad n \notin K \tag{3.19}$$

$$= \Delta_K \qquad if \qquad n \in K \tag{3.20}$$

Where the minimum number of controllable elements are denoted by K, which is any possible subset of the array elements N. No analytic solution is possible for this type of problem which is well suited for genetic algorithm solution. The formulation of such solution is given in chapter 4.

The nonlinear equation 3.18 has been solved using genetic algorithm. The use of genetic algorithm removes the restrictions on the size of array element displacement, required in the case of linear analytic method. The solution from the genetic algorithm provides the minimum number of elements to be selected along with their perturbations and locations. The element position perturbations $\hat{\Delta}_n$ are selected as the input variables to the genetic algorithm solution. This can be expressed as a chromosome vector 'V' of length N, where N is the number of array elements.

$$V = [\hat{\Delta}_1, \hat{\Delta}_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \hat{\Delta}_n]$$

Where $\hat{\Delta}_n$ is the position perturbation of element n, $n = 1, 2, \ldots\ldots, N$

$$\hat{\Delta}_n = 0 \qquad if \qquad n \notin K \tag{3.21}$$

$$= \Delta_K \qquad if \qquad n \in K \tag{3.22}$$

Where the minimum number of controllable elements are denoted by K, which is any possible subset of the array elements N.

Chapter 4

Array pattern nulling in Partially Adaptive Arrays using Genetic Algorithms

4.1 Introduction

Genetic algorithm has its roots in the process of natural evolution. In the early 1970's, john holland incorporated the features of natural evolution to yield a technique for solving difficult problems which he later named genetic algorithm. The algorithm manipulates the bit strings which he called chromosomes. A simulated evolution is carried out on the population of such chromosomes to find better chromosomes. Like in nature, the algorithm knows nothing about the type of problem it is solving. There are two mechanisms which link the genetic algorithm with the problem. One is the encoding of solutions to the problem with chromosomes and the other is the fitness function that measures the merit of any chromosome in the context of the problem.

No single encoding works best for all problems. Devising a good encoding is an important step in attacking a problem using genetic algorithm. Fitness function plays the same role in genetic algorithms that the environment plays in natural evolution. The interaction of an individual with its environment provides a measure of its fitness and the interaction of a chromosome with a fitness function provides a measure of fitness that the genetic algorithm uses when carrying out reproduction [11]. This chapter describes, the application of the genetic algorithm for array pattern nulling in partially adaptive arrays. The main contributions include

1. The fitness function is formulated that yields the nulls in the required direction and in the mean time it determines the optimum (minimum) number of controlled elements.

2. The adaptation space is chosen in a sequential fashion i.e the number of perturbed elements are increased one by one until the performance specifications are attained.

3. A sidelobe constraint is added by creating a pattern envelope to restrict the sidelobe variations.

This chapter is divided into four sections. The first section provides the introduction. The second section provides a step by step treatment of the functioning of the genetic algorithm. The third section, briefly describes the important steps involved in the developed genetic algorithm for partially adaptive array and the fourth section describes the main genetic algorithm.

4.2 Elements of the genetic algorithm

4.2.1 Population

Unlike most of the optimization techniques, genetic algorithm works on a number of encoded solutions (chromosomes) rather than on a single solution. An important decision to be made in this respect is the size of the population. The most effective population size is dependent on the problem being solved, the representation used, and the operators manipulating the representation. The initial population consisting of feasible solutions is generated randomly.

4.2.2 Fitness Function

The fitness function is used to assign a fitness value to each of the chromosomes and it is the only link between the physical problem being optimized and the general GA machinery. It takes a chromosome as an input and returns a number, which is a measure of the chromosome's performance on the problem to be solved. Fitness function plays the same role in genetic algorithm as the environment plays in natural evolution. Usually, the fitness value is the value of the fitness function or some scaled version of it. In general, the fitness function consists of the composition of two functions.

$$u(x) = g(f(x))$$

Where f is the fitness function and g transforms the value of the fitness function to a non-negative number.

4.2.3 Parent Selection

The parent selection is a preparatory step for the crossover operator. It chooses two parents (chromosomes) at a time to participate in the creation of a two new offsprings. Parent selection is usually based on fitness values, but it may take different forms. The simplest is the biased roulette wheel in which each individual in the population has a roulette wheel slot sized in proportion to its fitness. A simple spin of the roulette wheel yields an offspring. This scheme is a variation of stochastic sampling.

4.2.4 Crossover

Crossover is the main genetic operator. The crossover operator distinguishes genetic algorithm from all other optimization algorithms. It is performed on the chosen

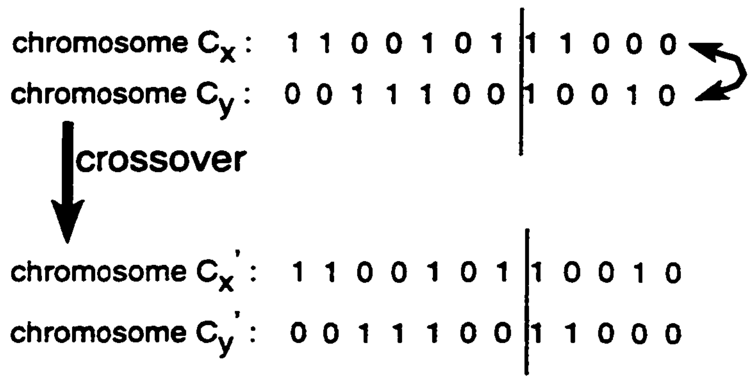


Figure 4.1: Crossover operation on the chromosome.

parents with a certain probabilities P_c. The operation of crossover results in the generation of two offsprings for some pairs of parents. The generated offsprings inherit some characteristics (good and/or bad) from both parents in a way similar to natural evolution. An example of this operator is depicted in figure 4.1. In this example we assume that each individual consists of a string of binary numbers. Given two strings (parents), a random cut point is chosen. Then, the offspring is generated by combining the segment of one parent to the left of the cut point with the segment of the other parent to the right of the cut point. This example shows a simple crossover operator. More sophisticated crossover operations are usually used on GA's. The overall progress of a GA is highly dependent on the choice of the crossover operation.

4.2.5 Mutation

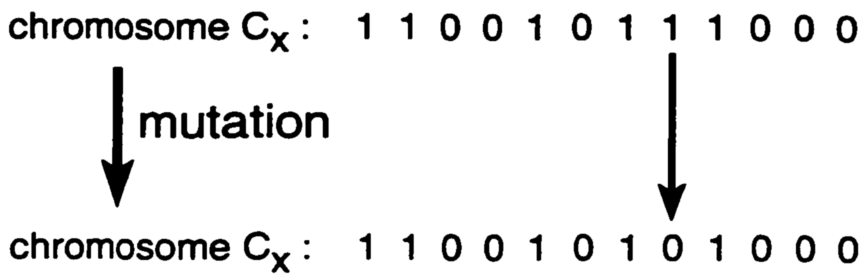


Figure 4.2: Mutation operation on the chromosome.

Mutation is the process of injecting new information in the population. It is a device for reintroducing diversity in to the population. It is a secondary operation which happens occasionally (with small probability P_m) to protect against the loss of important information. Furthermore, it helps in introducing some variations in the solutions in order to avoid getting trapped in local minima. The most popular mutation method is to select a random gene in a chromosome and to place a new random value in its place. For binary representation, it is the random alteration of a single position as shown in figure 4.2.

4.2.6 Selection

This operation is responsible for the selection of individuals for the new generation. It operates on the combined set of parents and offsprings. Three selection methods are most commonly used. These are competitive, random and stochastic.

In competitive selection, only the P fittest individuals are selected, where P is the population size. In random selection, the individuals are selected at random with uniform probability. Finally, in stochastic selection, an individual is selected with probability proportional to its fitness. It is similar to the roulette wheel described earlier for the selection of parents for crossover.

The overall picture of a GA is depicted in figure 4.3. Encoding is devised for a problem in hand. A population of encoded solution is created then fitness of each solution is found using fitness function. Two parents are selected using parent selection, for crossover which results in two offsprings. Offsprings are then mutated with a very low probability.

After the crossover is applied a specified number of times, we get a population of offsprings along with the old population of size P as shown in figure 4.4. A selection function is used to select individuals from these two population to get the new population of size P.

The above steps are then repeated Continuously. Highly fit individuals, or more precisely, highly fit characteristics, produce more copies of themselves in subsequent generation resulting in a general drift of the population as a whole towards an optimal solution point. The process can be terminated in several ways : threshold on the best individual (i.e., the process stops when an individual has an error less than some amount ε), number of generations exceeds a preselected value, or some other appropriate criteria. The best solution in the final population is the result of GA. To further materialize the concepts, the pseudo code of a genetic algorithm is given

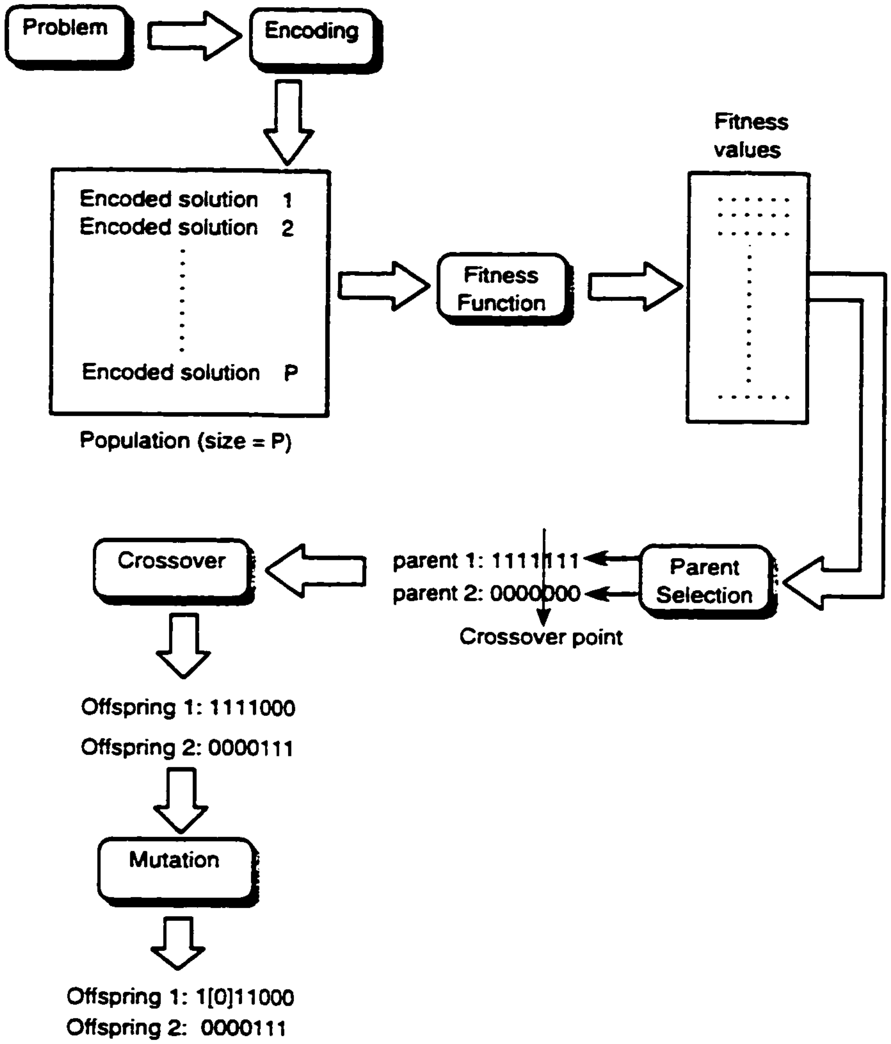


Figure 4.3: Genetic algorithm: Encoding, Fitness function, Parent Selection, Crossover and Mutation

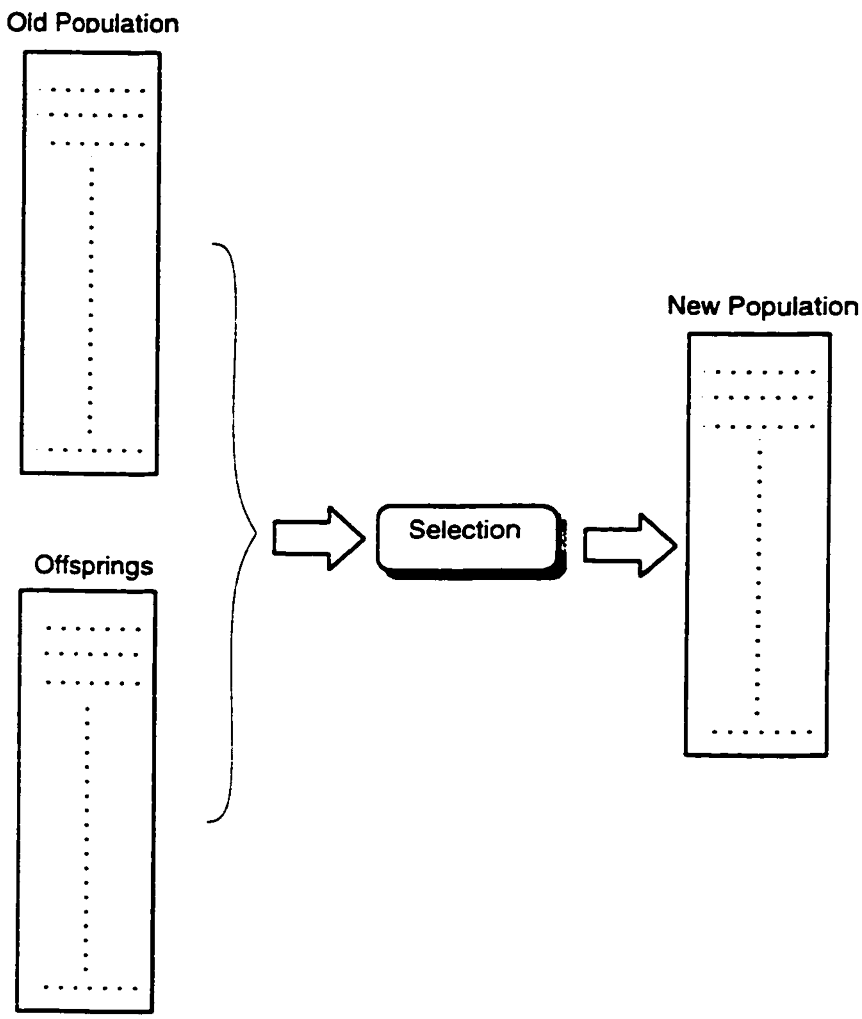


Figure 4.4: Genetic algorithm: selection

below.

Algorithm Simple GA
begin
Generate initial population and compute the Fitness of each chromosome;
while termination criteria is not met **do**
begin
Select parents from the current population;
Reproduce new population (offsprings) from these selected parents;
Evaluate the fitness of offsprings;
Replace current population by new population;
end;
end;

4.3 Important steps involved in the developed genetic algorithm for partially adaptive arrays

4.3.1 Initial Population

Generating an initial population is an important step in the design of GA's. In the case of fully adaptive arrays genetic algorithm works by generating a random initial population of real numbers within the specified limits, perturbing all the elements at a time, in all the randomly generated initial solutions (chromosomes). This is not feasible in the case of partially adaptive array, because in the partially adaptive arrays only the minimum number of elements are perturbed (controlled). Therefore in partially adaptive arrays GA starts by an initial population of solutions(chromosomes) with one controlled element only i.e. in all the initial population solutions (chromosomes) only a single element is perturbed with in the specified limits of $\pm 0.001\lambda$. This implies that the initial number of controlled elements (K) is set equal to 1.

4.3.2 Fitness function

Statement of the Problem :

Given an N element linear array with an interelement spacing of $\frac{\lambda}{2}$, we wish to steer a certain number of nulls "M" to known interference directions, by controlling the minimum number of elements "K" such that $1 < K < N$ and determine the locations of the K elements.

An optimization approach to such problem requires a multiobjective fitness function in which we have to specify the degree of importance of measures such as null depth and number of controlled elements. The main goal of the fitness function is to steer a null with at least a depth of -60 dB at desired locations. The secondary intent is to minimize the number of controlled elements.

The fitness function is formulated such that a null depth of -60 dB is achieved at the required directions, and at the same time, the number of controlled elements is minimized. These two goals are achieved by expressing the fitness function as the product of two objective functions $f_1 \times f_2$.

$$F = min[F(u_s) - F(u_m)]|_{m=1}^{m=M} \times [61 - K * C] \quad (4.1)$$

where $f_1 = min[F(u_s) - F(u_m)]|_{m=1}^{m=M}$ which ensure that the achieved null depth reaches -60 dB. where $F(u_s)$ is the array factor in the main beam direction, $F(u_m)$ is the array factor of the m's in the interfering signal directions and $f_2 = [61 - K * C]$ which minimizes the number of controlled elements. where K is the minimum number of controlled elements and C is an arbitrary constant, which is set initially to 60/N to give equal weight to both objective functions. The value of C can vary in order to increase or decrease the weight of the second objective function with respect to the first one. Once the GA subroutine, described in section 4.4, started it will stop if it achieves a null depth of -60 dB in the desired directions or exceeds a pre-defined number of generations. A flow chart of main program for partially adaptive arrays without sidelobe restrictions is shown in figure 4.5.

As the value of C increases, the weight of the second objective function increases with respect to the first one i.e. the allowed range of the controlled elements decreases and we get the required null depth in the desired directions, with lesser and lesser number of controlled elements, till a value of C is reached, with which we can get the required null depth with the lowest number of controlled elements. If the C

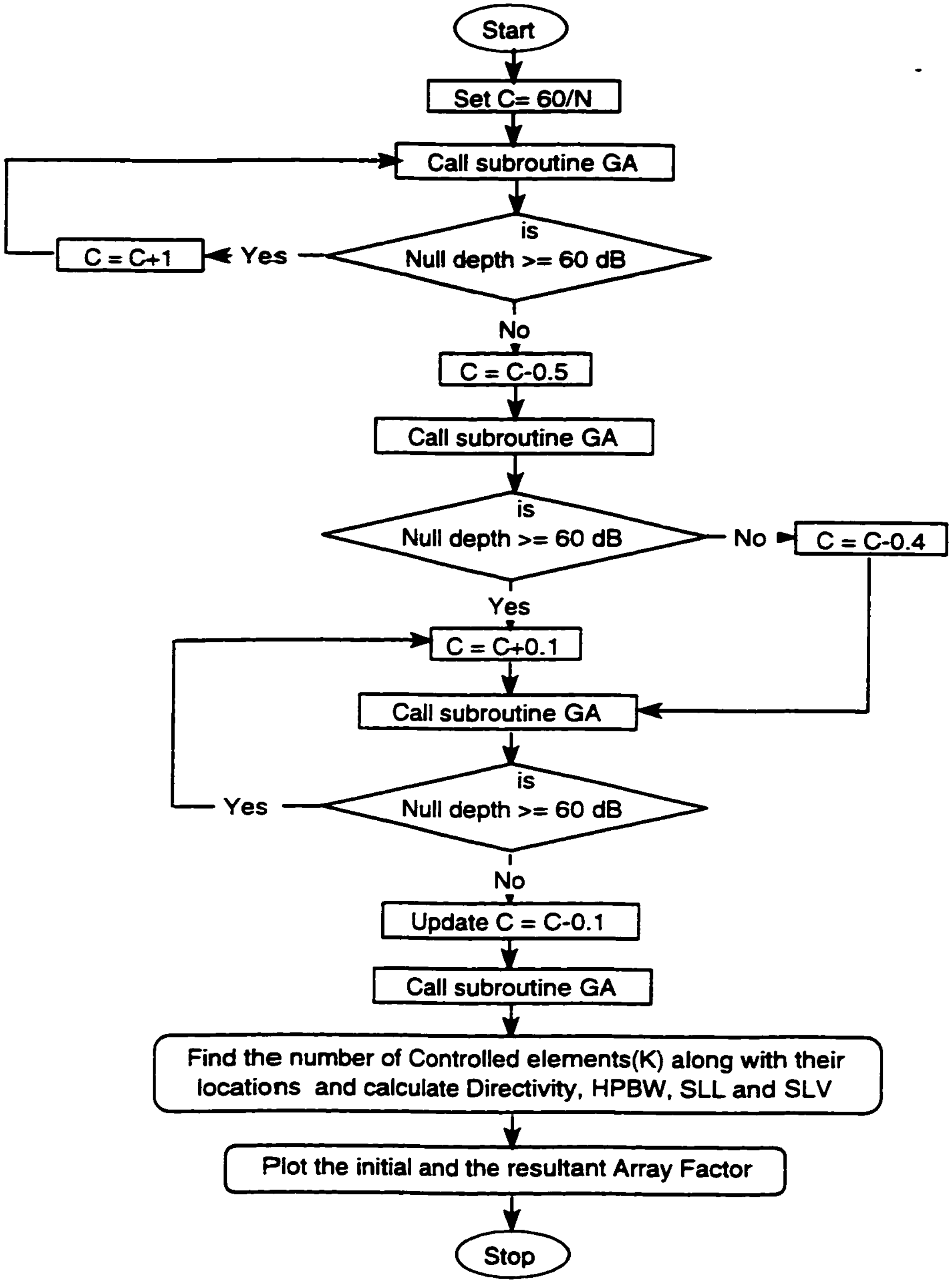


Figure 4.5: Flow chart of Main program for partially adaptive arrays without side-lobe restrictions.

is increased beyond this value the subroutine 'GA' will not converge. In other words, it will become impossible for the subroutine 'GA' to put a null depth of -60 dB in the desired directions with such a low number of controlled elements. Therefore for the given null directions, the convergence of the subroutine 'GA' at the highest possible value of C gives the minimum number of controlled elements.

4.3.3 Parent Selection

The parent selection is a preparatory step for the crossover operator. It chooses two parents (chromosomes) at a time to participate in the creation of a two new offsprings. In our case parent selection is based on biased roulette wheel scheme, in which each individual in the population has a roulette wheel slot sized in proportion to its fitness. A simple spin of the roulette wheel yields an offspring. This scheme is a variation of stochastic sampling. The probability of selecting an individual from the population is purely a function of the relative fitness of the individual. Individuals with high fitness will participate in the creation of the next generation more often then less fit individuals. The main advantage of this scheme is that there is still a finite probability that highly unfit individuals will participate in at least some of the matings, thereby preserving their genetic information.

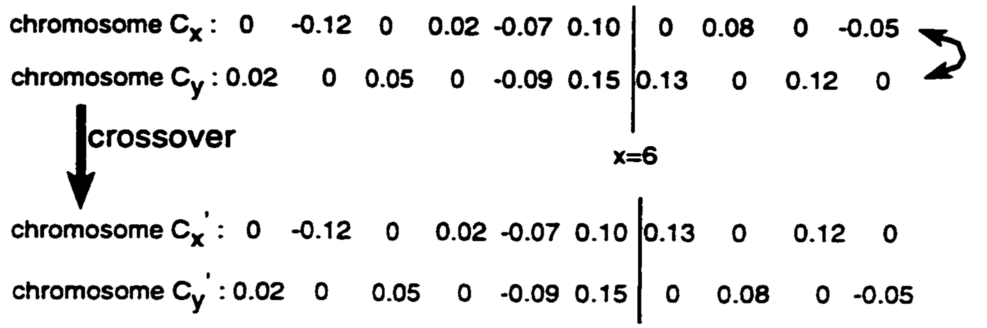


Figure 4.6: Crossover operation on the chromosome.

4.3.4 The Crossover Operator

A simple one point crossover operator is used. In this one point crossover operator a random location 'x' is generated over the length of the two parent chromosomes as a crossing point. The left part of one parent is combined with right part of other parent to generate offsprings as shown in figure 4.6.

Let us suppose that C_x and C_y be the chromosomes selected for the crossover. In other words, C_x and C_y are parents and will reproduce to give two offsprings, C'_x and C'_y. The crossover operator operates depending on the probability of crossover 'P_c' which is taken as 1 in our case, as this is found to be most effective.

4.3.5 The Mutation Operator

A simple single point mutation operator is used. The mutation is carried out by selecting a random gene in a chromosome and placing a new random value within

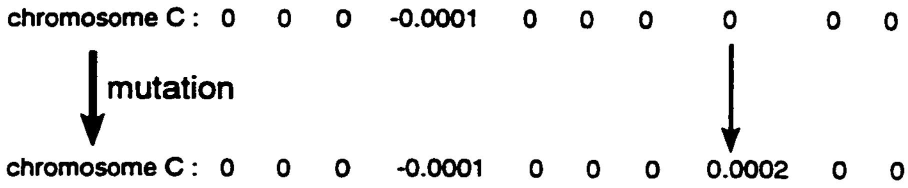


Figure 4.7: Mutation operation on the chromosome.

the specified limits of $\pm 0.001\lambda$, in its place as shown in figure 4.7 and this limit increases by $\pm 0.001\lambda$ per generation till $\pm 0.5\lambda$. The mutation operator is activated depending on the probability of mutation P_m which is taken as 0.5 in our case, much higher than for binary representations, as this is found to search the problem space more effectively.

This mutation operator serves two purposes in our problem. First, it increases the number of perturbed elements one by one in the chromosomes until the performance specifications are attained. Second, the side effect of using the crossover operator along with parent selection is that we might loose some potentially useful information from the chromosome. So, this mutation operator is used to avoid this side effect also.

4.3.6 Selection

Crossover is applied on the population with a specified probability P_c of 1. After all the crossovers are complete we get a double population consisting of initial population and offsprings then mutation is applied on this newly generated offsprings with a specified probability P_m of 0.5. We opted to have a fixed population size. Thus the next step is to transfer some of the individuals among initial population and offsprings to the next generation. This is done by a competitive selection method in which only the P fittest individuals are selected, where P is the population size. Initial population and offsprings are combined then their chromosomes are sorted in order according to their fitness values. Then best chromosomes are selected from this combination and a new population is created for the next generation. This scheme help in improving the search and maintaining the diversity in the population, which is necessary for search space exploration, and avoids premature convergence to the local optimum.

4.4 Description of the genetic Algorithm for null steering in partially Adaptive Arrays.

A computer program has been developed to determine the minimum number of controlled array elements, their position perturbations and locations required to impose the prescribed nulls in the array pattern and at the same time controls the sidelobe variation. The program is developed in MATLAB envirnoment and is based on genetic algorithms.

The program consists of four parts, one main program and three subroutines "GA", "fit" and "cross". The main genetic algorithm is implemented in subroutine "GA". The subroutines "fit" and "cross" are called in the subroutine "GA" when they are required and The subroutine "GA" is called in the main program as shown in figure 4.5. The subroutine fit calculates the fitness of chromosomes and the subroutine cross is used to cross the parents to produce offsprings.

In our problem the element position perturbations are selected as the input variables to the genetic algorithm solution. Each chromosome is effectively a vector of the genes g_1 to g_N, N is the number of variables. The gene g_i represents the perturbation of the i^{th} element.

The genes are coded as floating point numbers so as to increase the computational efficiency and precision of the algorithm. These floating point numbers are constrained to the range $\pm 0.001\lambda$ and increases by $\pm 0.001\lambda$ per generation till $\pm 0.5\lambda$. The probability of crossover P_c is taken as 1 and The probability of mutation P_m is taken as 0.5, much higher than for binary representations, as these are found to search the problem space more effectively.

The subroutine "GA" starts with a given value of an arbitrary constant C used in the fitness function described in section 4.3.2 and then initializes the parameters such as the population size 'P', the length of the chromosome 'N', the probability of crossover P_c, the probability of mutation P_m etc. The population size 'P' refer's to the number of randomly generated different solutions to our problem.

The flow chart of the subroutine "GA" is shown in figure 4.8. After initializing the parameters GA generates an initial population of P chromosomes with single controlled element i.e. initially solutions are generated, perturbing (controlling) only a single element. The fitness function described in section 4.3.2 is then evaluated for each of the chromosomes in the population by calling a subroutine called "fit". The flow chart of the subroutine "fit" is shown in figure 4.9. The population size used in this case is 100 as this is found to search the problem space more effectively.

The parents are then selected from these populations using a roulette wheel selection method. Crossover is then applied by calling a subroutine "cross" based on a single point crossover operator, resulting in a pair of offsprings. The flow chart of the subroutine "cross" is shown in figure 4.10.

This process continues until we get a new population of offsprings. A single point mutation operator is then applied. The fitness function described in section 4.3.2 is then evaluated for each chromosome of the newly generated population of offsprings using the subroutine "fit".

The newly generated offsprings are sorted along with the initial population depending upon their fitness values and the fittest 'P' chromosomes are selected from this combination. Among these 'P' chromosomes the minimum null depth of the best chromosome is compared with the desired value i.e a null depth of -60dB. If it is greater than -60 dB then these best 'P' chromosomes obtained are taken as the new population for the next generation.

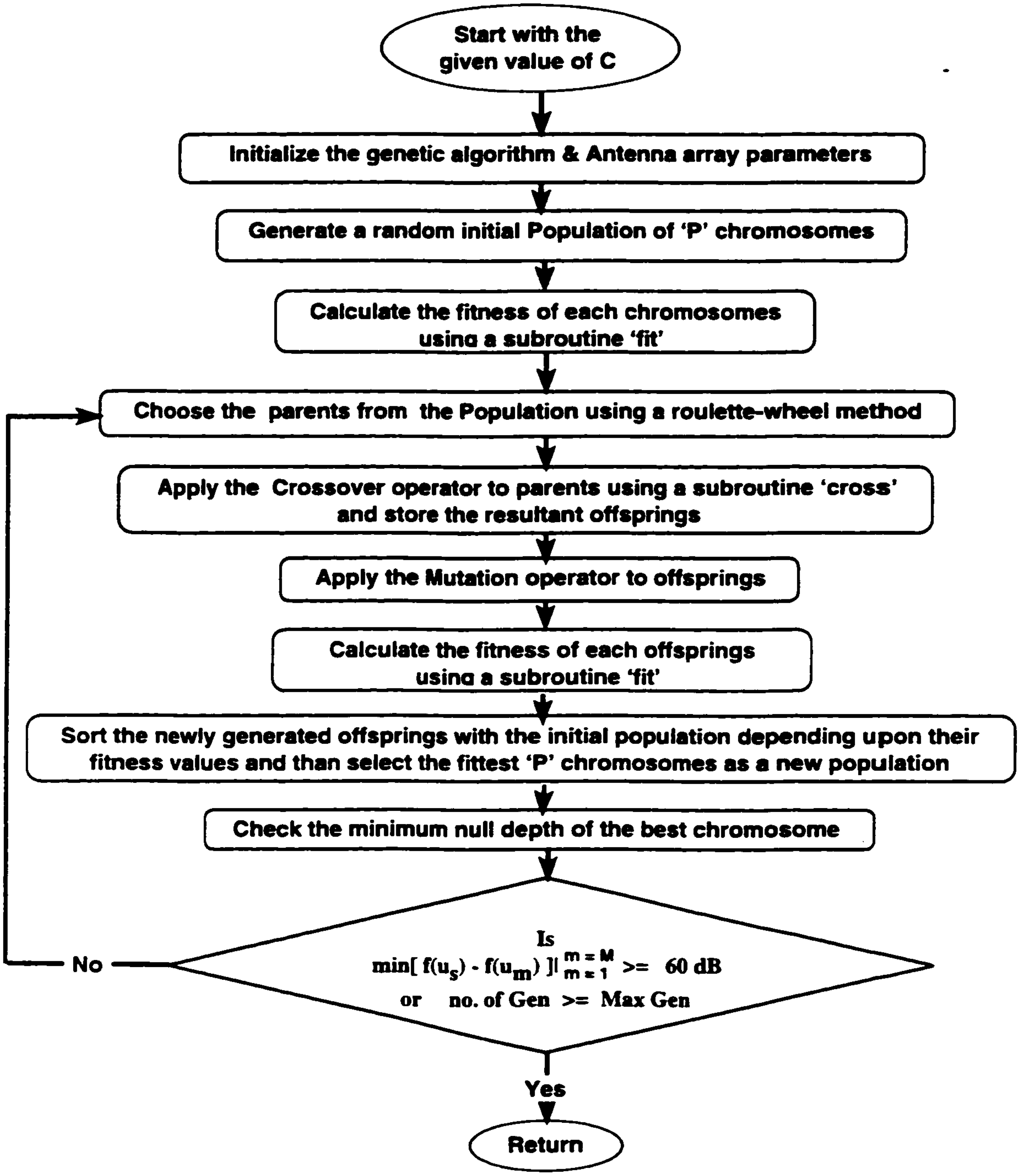


Figure 4.8: Flow chart of the subroutine GA.

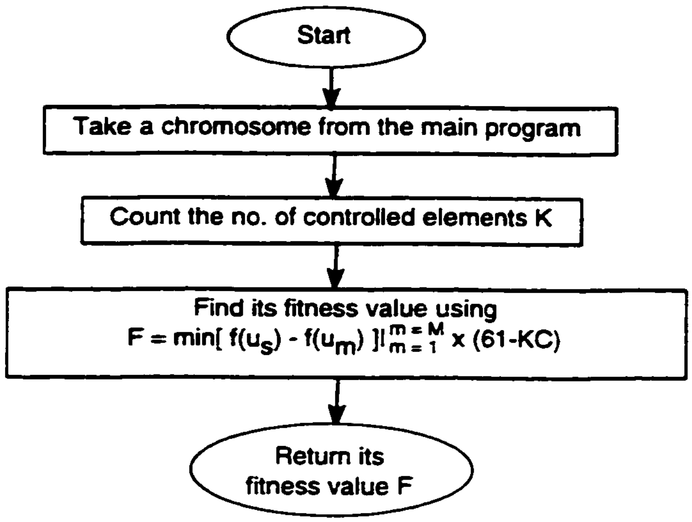


Figure 4.9: Flow chart of the subroutine 'fit'.

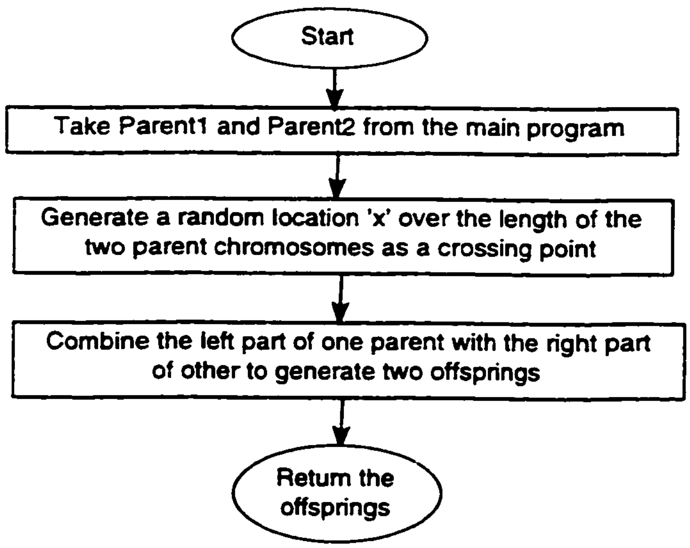


Figure 4.10: Flow chart of the subroutine 'cross'.

This process continues till all the nulls were at or below -60 dB or the number of generations exceeds a pre-defined number. The stopping criteria used here is null depth and not the minimum number of controlled elements because null depth is defined but we don't know the minimum number of controlled elements. An arbitrary constant C in the fitness function described in section 4.3.2 determines the minimum number of controlled elements. A flow chart of the main program for partially adaptive arrays without sidelobe restrictions is shown in figure 4.5. For the given null directions, the convergence of the subroutine GA at the highest possible value of C gives the minimum number of controlled elements.

Initially the GA has been implemented in order to steer the nulls to the required directions without putting constraints on the sidelobe variations. With this implementation we are able to minimize the number of controlled elements to a great extent but at the cost of high sidelobe variations. Therefore a second implementation of the GA has been devised in order to minimize the sidelobe variations.

4.4.1 Fitness function implementation with sidelobe constraints

In this implementation the subroutine "fit", which is used to evaluate fitness of chromosomes includes an extra constraint for checking the sidelobe variation. In the case of uniform arrays this Constraint is added by creating a step pattern envelope with respect to initial pattern as shown in the figure 4.11. The sidelobe variation (SLV) is defined as the maximum positive difference between this step pattern envelope and the perturbed pattern as shown in figure 4.11. In the case of chebyshev arrays, the sidelobe variation (SLV) is defined as the maximum positive difference between the sidelobe level and the perturbed pattern. The flow chart of the subroutine "fit" for this case is shown in figure 4.12.

In this case of with sidelobe restrictions, before evaluating the fitness function of each chromosome the subroutine fit first calculates its array pattern values at all angles and compares these values with the specified sidelobe limits. If the maximum difference between these values is less than the allowed sidelobe variation (SLV) then it proceeds to calculate the fitness value using the fitness function described in section 4.3.2. If these values are greater than the allowed sidelobe variation (SLV) then it gives a lowest fitness value of 1 for that chromosome such that it gets rejected automatically in the main program i.e. the subroutine "fit" allows only those chromosomes which satisfies the allowed sidelobe variation SLV limit. The population size used in this case is 300. This increase in population size is due to the addition of the sidelobe constraint.

In this case, An arbitrary constant C in the fitness function described in section 4.3.2 and the allowed SLV are selected such that, we get the desired null depths in the prescribed directions with minimum number of controlled elements and SLV. A

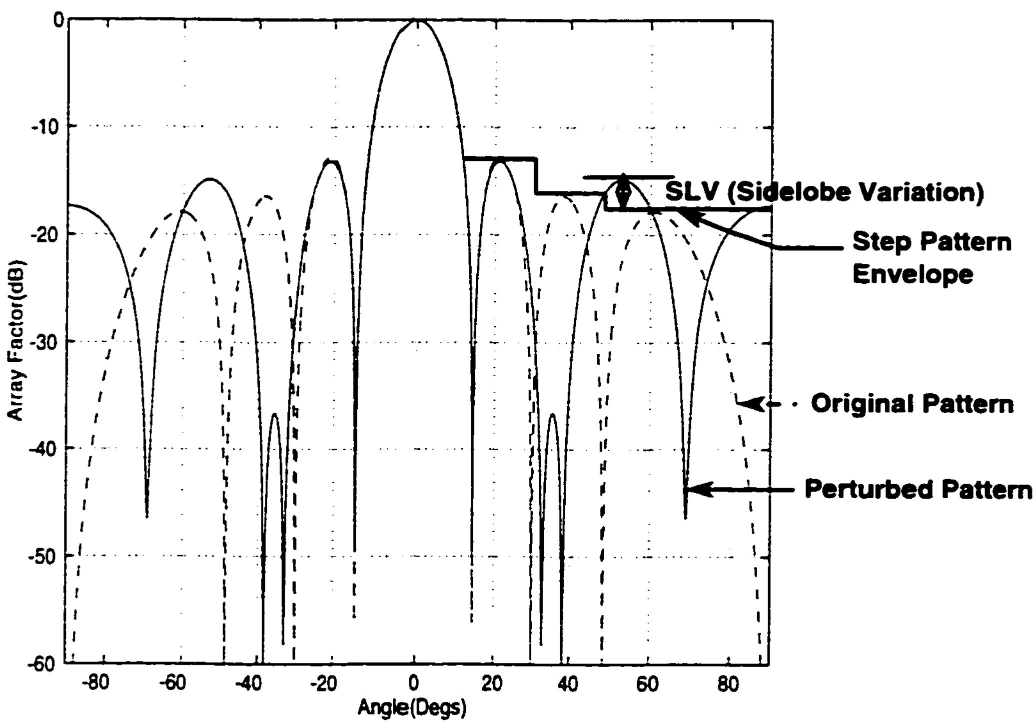


Figure 4.11: Figure showing the sidelobe variation SLV

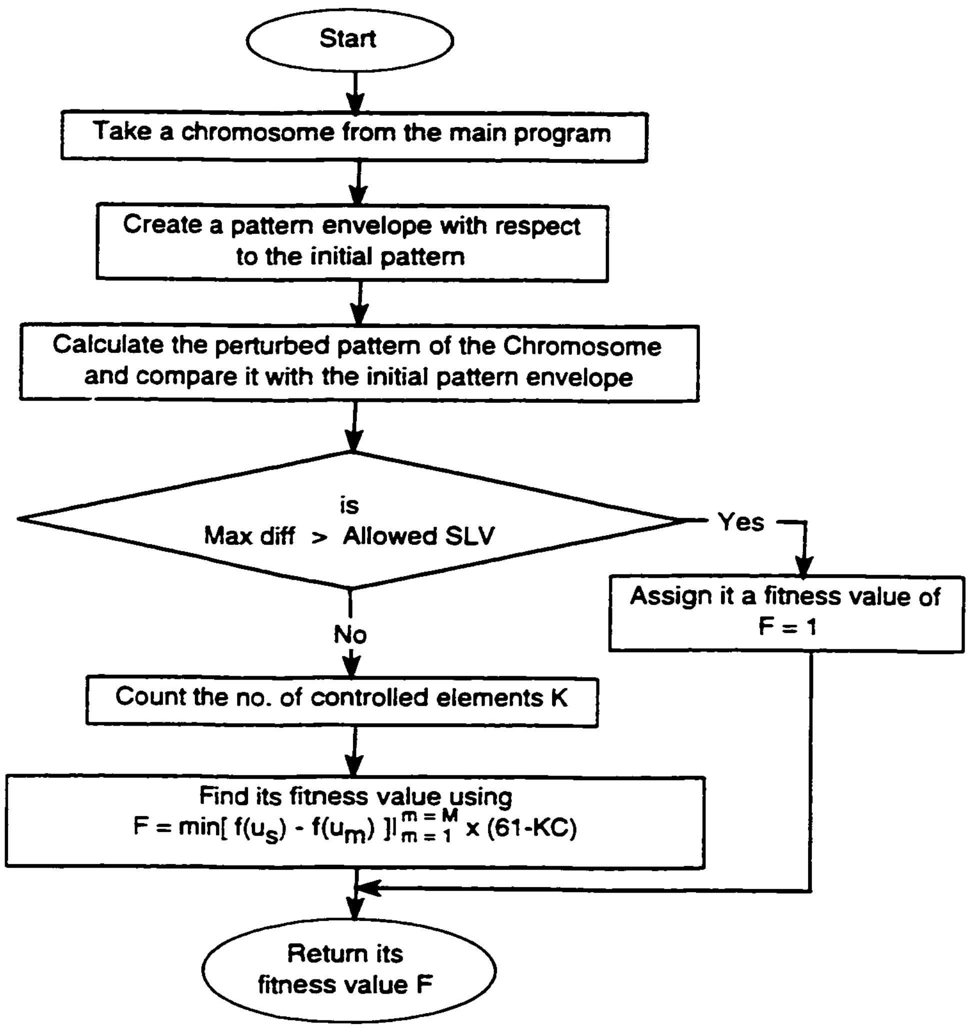


Figure 4.12: Flow chart of subroutine "fit" for partially adaptive arrays with sidelobe restriction.

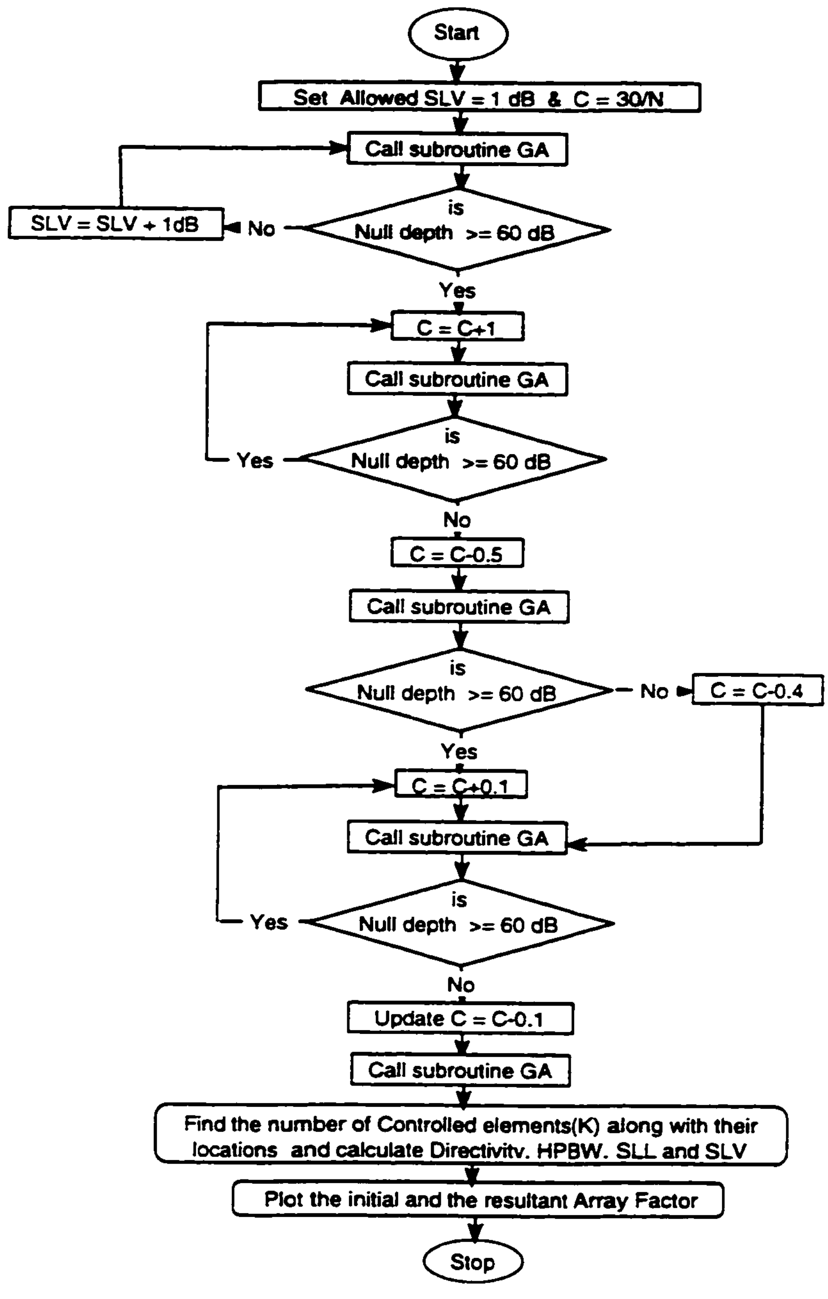


Figure 4.13: Flow chart of Main program for partially adaptive arrays with sidelobe restrictions.

flow chart of main program for partially adaptive arrays with sidelobe restrictions is shown in figure 4.13. For the given null directions, the convergence of the subroutine GA at the highest possible value of C with the lowest possible value of SLV gives the minimum number of controlled elements with the minimum SLV.

Chapter 5

Results and Computer Simulations

5.1 Introduction

In this chapter we present the simulation results of null steering in partially adaptive arrays using the developed algorithm of chapter 4. The developed program is tested to validate the capability of null steering by controlling a minimum number of elements.

The simulations were done for partially adaptive arrays with 8, 16 and 30 isotropic elements of half wavelength element spacing using uniform and Dolph-chebyshev current distributions. One further sidelobe constraint is added by creating a pattern envelope in both uniform and chebyshev arrays to reduce the sidelobe variation.

The results are demonstrated by steering one to six nulls. This is followed by fixing the number of controlled elements so that the imposed nulls could be scanned over the entire sidelobe region after the first sidelobe.

The performance of 8 element uniform and chebyshev partially adaptive arrays are compared with that of the fully adaptive array, when imposing nulls at specified directions in sections 5.2 and 5.3 respectively. The study is repeated when sidelobe constraints are applied. Determination of the realizable minimum no. of controlled elements has been found and tested for both uniform and chebyshev array in sections 5.2.3 and 5.3.3 respectively.

The performance of 16 element uniform and chebyshev partially adaptive arrays are compared with that of the fully adaptive array, when imposing nulls at specified directions in sections 5.4 and 5.5 respectively. The study is repeated when sidelobe constraints are applied. Determination of the realizable minimum no. of controlled elements has been found and tested for both uniform and chebyshev array in sections 5.4.3 and 5.5.3 respectively.

The performance of 30 element uniform and chebyshev partially adaptive arrays are compared with that of the fully adaptive array, when imposing nulls at specified directions in sections 5.6 and 5.7 respectively. The study is repeated when sidelobe constraints are applied. Determination of the realizable minimum no. of controlled elements has been found and tested for both uniform and chebyshev array in sections 5.6.3 and 5.7.3 respectively.

The following characteristics of partially controlled arrays has been investigated.

1. The effect of reducing the number of controlled elements on the array parameters, such as the half-power beamwidth (HPBW), Directivity and Sidelobe level (SLL).
2. The effect of steering a null towards the main beam on the minimum number of controlled elements.
3. The effect of adding a sidelobe constraint on the minimum number of controlled elements.
4. The effect of varying the number of nulls on the array performance.

5.2 Simulation results of 8 element uniform partially adaptive arrays

5.2.1 Introduction

In this section, we study the array performance and behavior of its parameters such as half-power beam width (HPBW), directivity , sidelobe level (SLL), the minimum number of controlled elements K and the sidelobe variation (SLV) on 8-element uniform array with element spacing of 0.5 λ.

The validity of the proposed partially adaptive method is examined by first placing a single null on the peak of each of the sidelobes separately and then placing two nulls on the peak of first and second sidelobes. The results are compared with the fully adaptive case with and without sidelobe restrictions.

5.2.2 Simulation results

The results of Fig.5.1 show one null which has been steered to the peak of the first sidelobe level at 21.1°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.1(a) when all the 8-elements are perturbed without restricting the sidelobe variation. Fig.5.1(b) shows the perturbed pattern compared to the initial, when all the 8-elements are perturbed and the sidelobe variation is restricted to 1.79 dB. Fig.5.1(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=2 in this case. The required null has been achieved precisely, but the sidelobe level variation has changed to more than 10dB. Fig.5.1(d) shows the pattern when the sidelobe variation is restricted to 2.05 dB while achieving the required null. K is equal to 4 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.1 are given in table 5.1 and table 5.2 respectively.

The results of Fig.5.2 (a) and (b) show one null, which has been steered to the peak of the second sidelobe level at 38.2° and the results of Fig.5.2 (c) and (d) show one null, which has been steered to the peak of the third sidelobe level at 60.8°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.2(a)

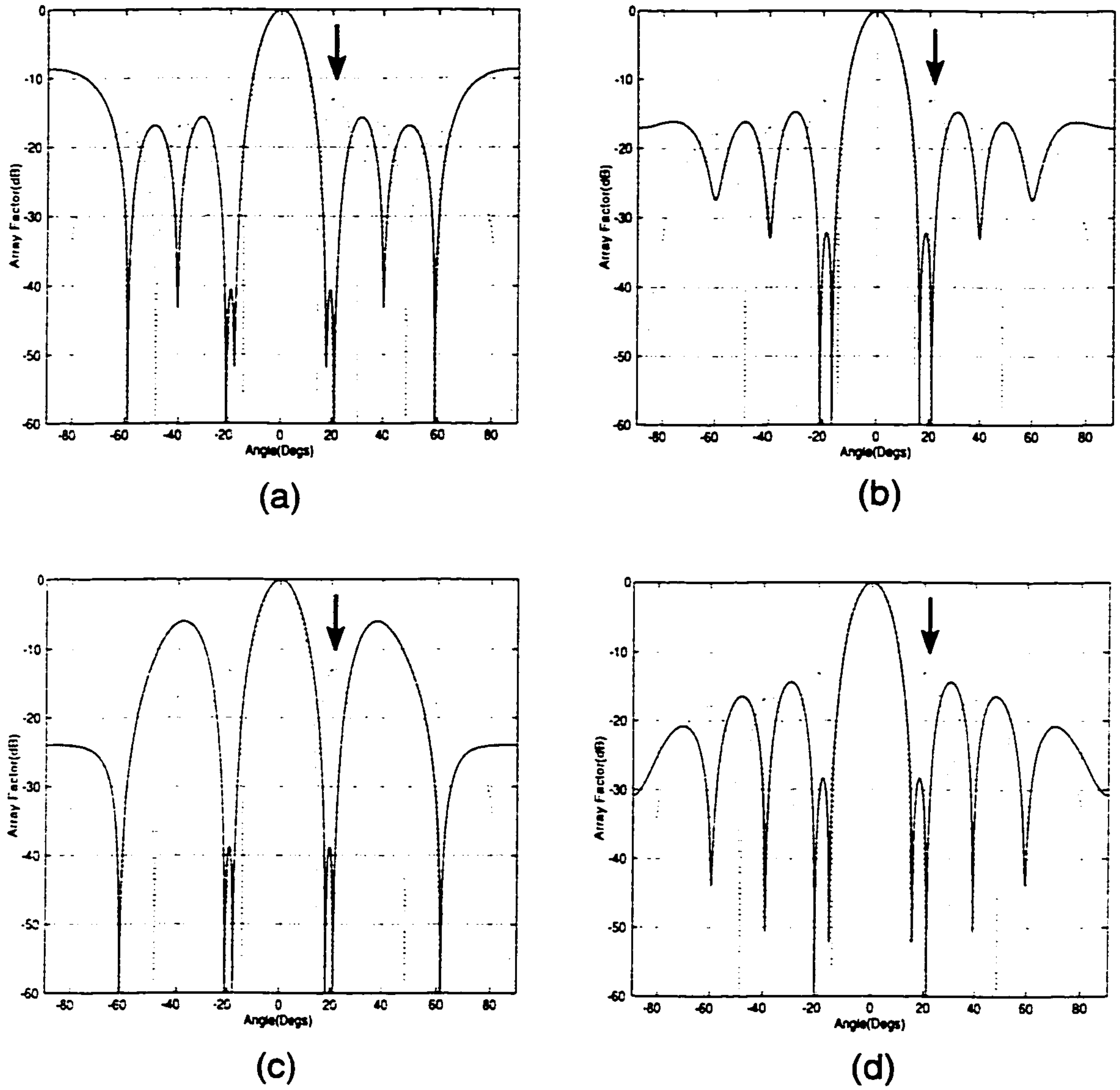


Figure 5.1: Array patterns of 8 element uniform array with one null imposed on the peak of first sidelobe at 21.1° (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 1.76 dB (c)Minimum (optimum) number of elements are controlled (K=2) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=4) with sidelobes restricted to 2.05 dB

ELEMENT Number	Fig. 5.1(a) Full(WOSR) Δ_n	Fig. 5.1(b) Full(WSR) Δ_n	Fig. 5.1(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.1(d) Partial(WSR) $\hat{\Delta}_n$
1	-0.1448	-0.2097	0	-0.2651
2	0.1516	0.0715	0	0
3	0.1548	0.1752	0.4575	0.1794
4	0.1220	0.0409	0	0
5	-0.1190	0.0007	0	0
6	-0.1660	-0.2045	-0.4608	-0.1695
7	-0.1561	-0.0472	0	0
8	0.1578	0.2206	0	0.2612

Table 5.1: Computed element position perturbations for Fig. 5.1 when a single null is steered at 21.1°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.1(a) Full(WOSR)	Fig. 5.1(b) Full(WSR)	Fig. 5.1(c) Partial(WOSR)	Fig. 5.1(d) Partial(WSR)
Min. Controlled Elements K		8	8	2	4
SLV (dB)		9.2477	1.7660	10.4326	2.0575
DIRECTIVITY	8	7.6207	8.2858	5.6611	8.5754
HPBW (DEG.)	12.8025	13.0545	12.4847	13.5642	12.0888
SLL (dB)	-12.8	-8.6429	-14.6880	-5.9952	-14.3703
Null Depth(dB)	-12.8	-60.42	-62.45	-62.44	-60.49
No. of Generations		182	1115	560	279
CPU time (Sec)		60.66	4460	186	1116

Table 5.2: Computed Array Parameters for Fig. 5.1 when a single null is steered at 21.1°

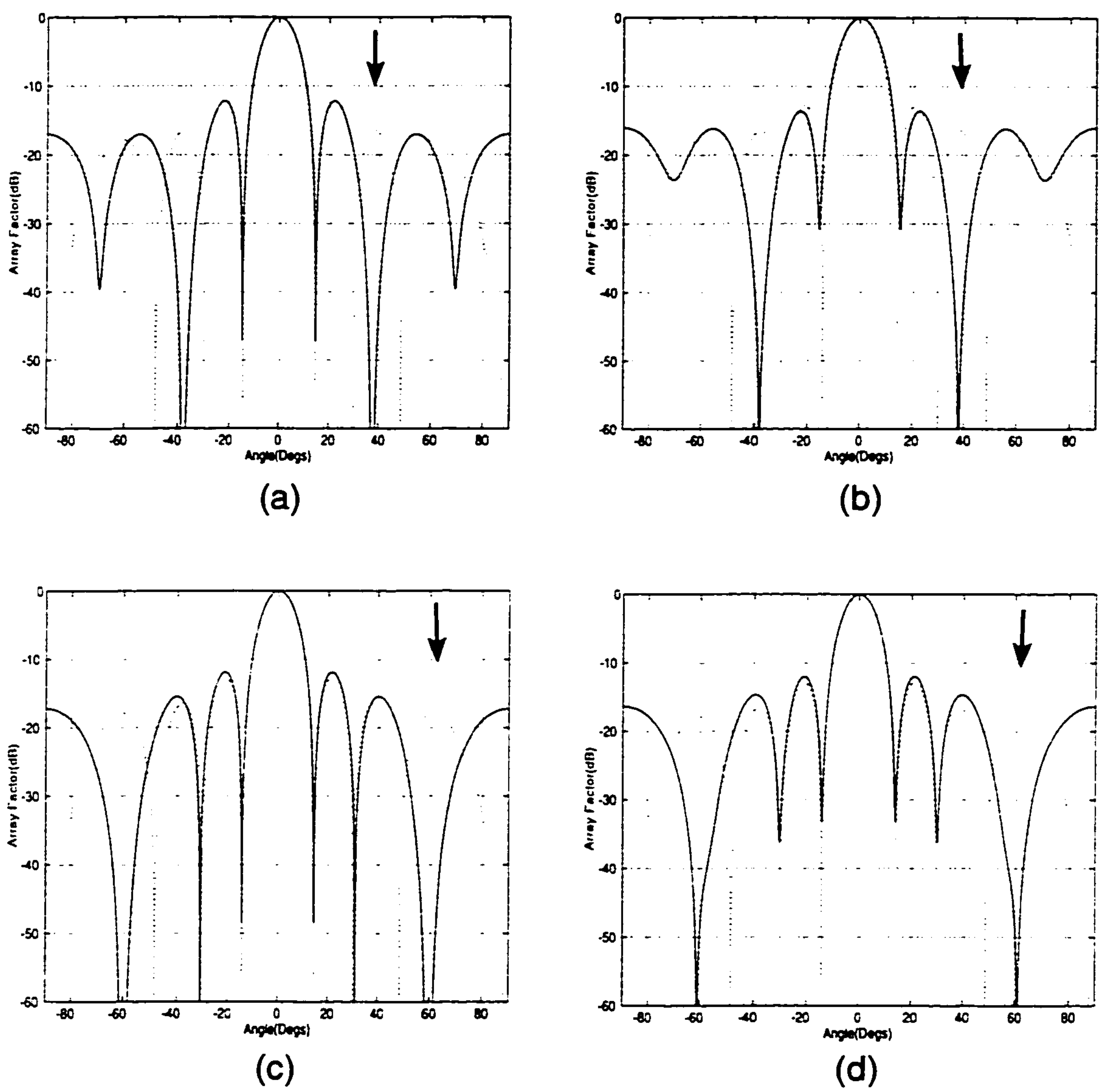


Figure 5.2: Array patterns of 8 element uniform array with (a) one null imposed on the peak of second sidelobe at 38.2° when all elements are controlled and sidelobes restricted to 0.89 dB (b) Minimum (optimum) number of elements are controlled (K=3) with sidelobes restricted to 1.86 dB (c) one null imposed on the peak of third sidelobe at 60.8° when all elements are controlled and sidelobes restricted to 1 dB (d) Minimum (optimum) number of elements are controlled (K=2) with sidelobes restricted to 1.76 dB

when all the 8-elements are perturbed and the sidelobe variation is restricted to 0.89 dB. Fig.5.2(b) shows the pattern when the sidelobe variation is restricted to 1.86 dB while achieving the required null. K is equal to 3 in this case. Fig.5.2(c) shows the resulting pattern when all the 8-elements are perturbed and the sidelobe variation is restricted to 1 dB. Fig.5.2(d) shows the pattern when the sidelobe variation is restricted to 1.76 dB while achieving the required null. K is equal to 2 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.2 are given in table 5.3 and table 5.4 respectively.

In general for the case of a single null in 8-element uniform partially adaptive

ELEMENT Number	Fig. 5.2(a) (Null at 38.2°) Full(WSR) Δ_n	Fig. 5.2(b) (Null at 38.2°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.2(c) (Null at 60.8°) Full(WSR) Δ_n	Fig. 5.2(d) (Null at 60.8°) Partial(WSR) $\hat{\Delta}_n$
1	-0.0049	0	0.0266	0
2	0.0954	0.1022	0.0248	0
3	-0.0429	0	-0.0565	-0.1203
4	-0.0510	0	0.0347	0
5	0.0453	0	-0.0415	-0.0777
6	0.0356	0	0.0504	0
7	-0.1236	-0.2726	-0.0229	0
8	-0.0101	-0.0981	-0.0258	0

Table 5.3: Computed element position perturbations for Fig. 5.2 as a function of λ.

arrays with sidelobe restriction, it is observed that as we steer a single null towards the main beam from the peak of third sidelobe to the peak of first sidelobe, the minimum no. of controlled elements K and the SLV increase. The reason for this

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.2(a) (Null at 38.2°) Full (WSR)	Fig. 5.2(b) (Null at 38.2°) Partial (WSR)	Fig. 5.2(c) (Null at 60.8°) Full (WSR)	Fig. 5.2(d) (Null at 60.8°) Partial (WSR)
Min. Controlled Elements K		8	3	8	2
SLV (dB)		0.8922	1.8610	0.9880	1.7657
DIRECTIVITY	8	7.8358	7.7455	7.8370	7.9536
HPBW (DEG.)	12.8025	13.0389	13.6299	13.0389	12.6871
SLL (dB)	-12.8	-12.2079	-13.5996	-11.8775	-11.9703
Initial Null Depth(dB)		-16.42	-16.42	-17.89	-17.89
Final Null Depth(dB)		-72.08	-72.23	-75.49	-60.00
No. of Generations		454	426	687	205
CPU time (Sec)		1816	1704	2748	820

Table 5.4: Computed Array Parameters for Fig. 5.2

behavior is that as we are steering a null from a lower energy concentrated area to a higher energy concentrated area, it requires a higher order of degrees of freedom. It is also observed that, when the sidelobe restriction is applied, the number of controlled elements increase from the minimum possible value. This is due to increasing the number of constraints in this optimization problem. Tables 5.2 and 5.4, show a comparison of array parameters such as directivity, HPBW and SLL. The results show that they are almost unchanged. The sidelobe variation (SLV) in case of partially adaptive arrays is slightly higher than in fully adaptive case, since it is obvious that the best array parameters are obtained, when all the 8-elements are perturbed, as it affords the greatest control over the array response.

The results of Fig.5.3 show two nulls, which has been steered to the peaks of the first and second sidelobe levels at 21.1° and 38.2° respectively. The perturbed

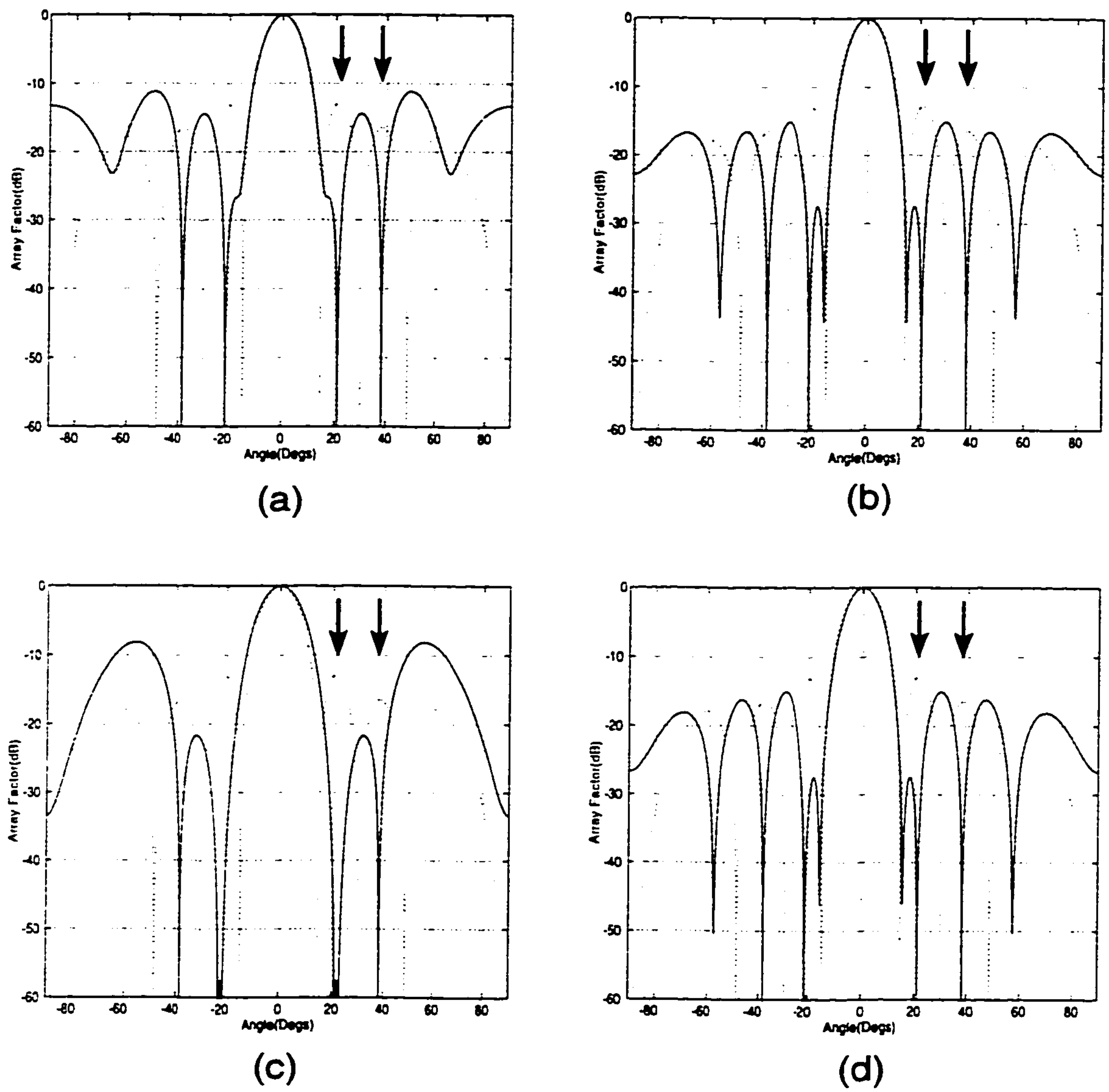


Figure 5.3: Array patterns of 8 element uniform array with two nulls imposed on the peak of first and second sidelobes at 21.1° and 38.2° (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 1.36 dB (c)Minimum (optimum) number of elements are controlled (K=4) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=6) with sidelobes restricted to 1.69 dB

pattern compared to the initial pattern (dotted) is shown in Fig.5.3(a) when all the 8-elements are perturbed without restricting the sidelobe variation. Fig:5.3(b) shows the perturbed pattern compared to the initial, when all the 8-elements are perturbed and the sidelobe variation is restricted to 1.36 dB. Fig.5.3(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=4 in this case. The required nulls has been achieved precisely, but the sidelobe level variation has changed to more than 9 dB. Fig.5.3(d) shows the pattern when the sidelobe variation is restricted to 1.69 dB while achieving the required nulls. K is equal to 6 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.3 are given in table 5.5 and table 5.6 respectively.

For the case of two nulls in 8-element uniform partially arrays with sidelobe re-

ELEMENT Number	Fig. 5.3(a) Full(WOSR) Δ_n	Fig. 5.3(b) Full(WSR) Δ_n	Fig. 5.3(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.3(d) Partial(WSR) $\hat{\Delta}_n$
1	-0.2415	-0.2909	0	-0.2923
2	0.2019	0.0113	0.3311	0
3	0.3439	0.1564	0.2020	0.1550
4	0.0699	0.0274	0	0.0127
5	0.2389	-0.0047	0	0
6	-0.1355	-0.1269	-0.2063	-0.1426
7	0.1368	0.0194	-0.3297	0.0118
8	0.2336	0.2832	0	0.2809

Table 5.5: Computed element position perturbations for Fig. 5.3 when two nulls are steered at 21.1° and 38.2°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.3(a) Full(WOSR)	Fig. 5.3(b) Full(WSR)	Fig. 5.3(c) Partial(WOSR)	Fig. 5.3(d) Partial(WSR)
Min. Controlled Elements K		8	8	4	6
SLV (dB)		6.7496	1.3631	9.7421	1.6918
DIRECTIVITY	8	8.2338	8.6490	6.3035	8.6707
HPBW (DEG.)	12.8025	12.17	11.9491	14.4129	11.9514
SLL (dB)	-12.8	-11.1410	-15.0721	-8.1485	-15.0284
Null Depth(dB)	-12.8	-61.37	-60.19	-60.63	-61.26
No. of Generations		2510	1729	615	418
CPU time (Sec)		836.66	6916	205	1672

Table 5.6: Computed Array Parameters for Fig. 5.3 when two nulls are steered at 21.1° and 38.2°.

striction, it is observed that as we increase the number of nulls from one to two, the minimum no of controlled elements K and SLV has also increased. From the table 5.6 comparing the array parameters of partially adaptive array, with the corresponding fully adaptive array, when two nulls are imposed at first and second sidelobes, one notices that directivity, HPBW, SLL are almost unchanged. The sidelobe variation (SLV) in case of partially adaptive arrays is slightly higher than in fully adaptive case. since it is obvious that the best array parameters are obtained, when all the 8-elements are perturbed, as it affords the greatest control over the array response.

5.2.3 Determination of the realizable minimum number of controlled elements

From the results obtained, it is found that the minimum number K and the location of the controlled elements in partially adaptive arrays depend on the number and location of nulls. In other words , the minimum number K and location of the controlled elements for partially adaptive method vary with the number and location of imposed nulls. This is not desirable in element position perturbation technique, since it requires to install a motor for each element and hence the number of motors are not being reduced. Therefore, this technique can only be realistically implemented if a limited number of motors are fixed at some optimum elements locations of an array capable of providing a single or multiple imposed nulls at locations covering most of the sidelobe region, while other array parameters are nearly kept unchanged.

To implement this technique for a single null, the null location which requires the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steering. In the case of 8-element uniform partially adaptive arrays a null on the peak of first sidelobe requires the use of 4 controlled elements. Now keeping the location of these 4 controlled elements fixed, a single null is imposed on the peak of second sidelobe as shown in Fig.5.4(b) and a single null is imposed on the peak of third sidelobe as shown in Fig.5.4(c). Therefore perturbing only these 4 fixed elements out of 8 elements, we can steer a single null any where in the sidelobe region of an 8-element uniform partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.4 are given in table 5.7 and table 5.8 respectively.

The results given in table 5.8 show that the sidelobe variation (SLV) when the null

ELEMENT Number	Fig. 5.4(a) (Null at 21.1°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.4(b) (Null at 38.2°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.4(c) (Null at 60.8°) Partial(WSR) $\hat{\Delta}_n$
1	-0.2651	-0.2026	-0.2700
3	0.1794	0.0504	0.1361
6	-0.1695	-0.0424	-0.1315
8	0.2612	0.2321	0.2151

Table 5.7: Computed element position perturbations for Fig. 5.4 as a function of λ.

is imposed at second or third sidelobe, is higher than the previous results inspite of increasing the no. of controlled elements. This shows that for a particular null location, the SLV depends on the location of the minimum controlled elements. The 8-element uniform partially adaptive array is not suitable for steering two or more nulls, because steering two nulls requires a minimum of 6 controlled elements out of 8 elements.

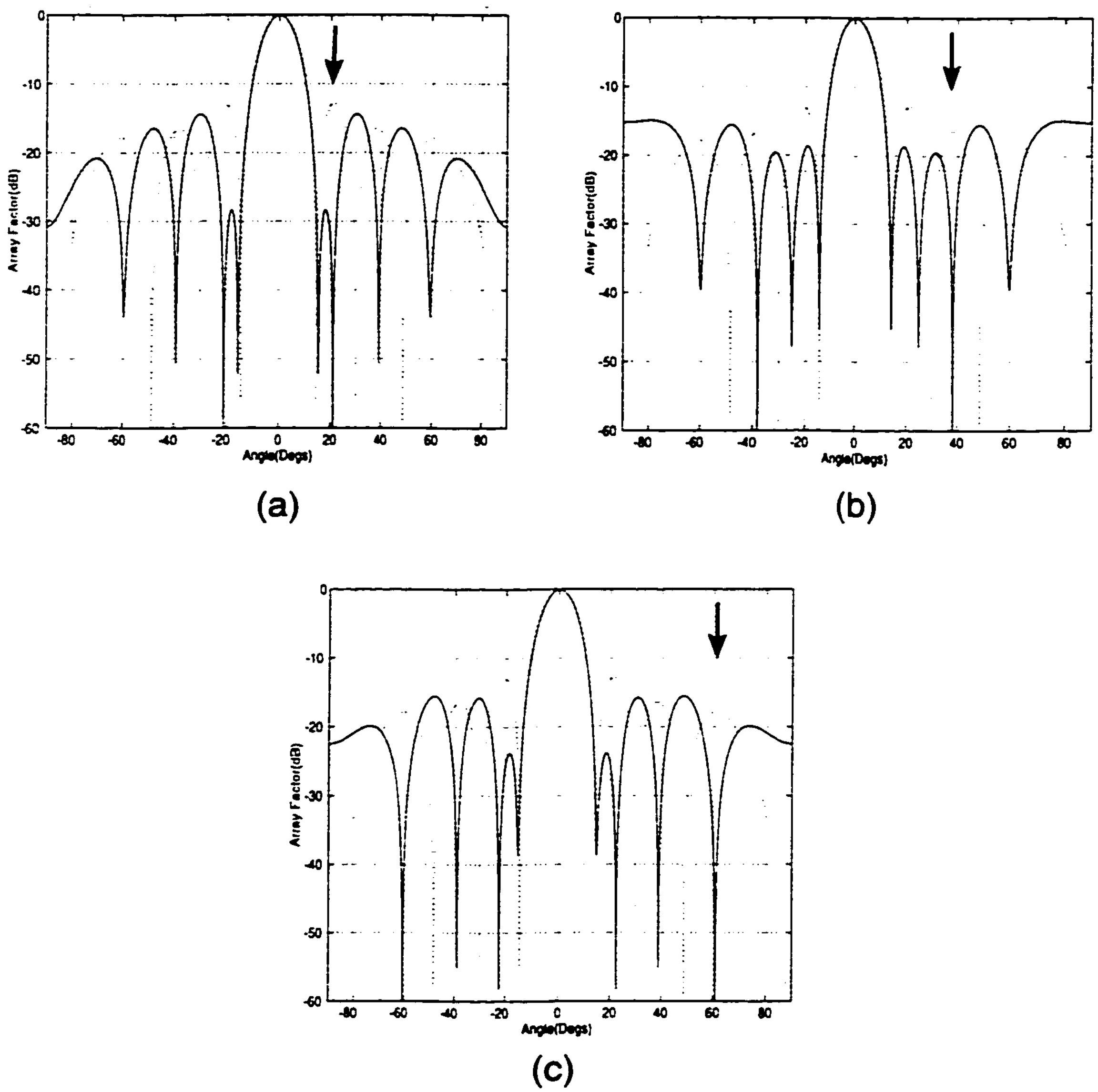


Figure 5.4: Array patterns of 8 element uniform array perturbing 4 fixed elements with (a) one null imposed on the peak of first sidelobe at 21.1° and sidelobes restricted to 2.05 dB (b) one null imposed on the peak of second sidelobe at 38.2° and sidelobes restricted to 2.95 dB(c)one null imposed on the peak of third sidelobe at 60.8° and sidelobes restricted to 2.33 dB

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.4(a) (Null at 21.1°) Partial(WSR)	Fig. 5.4(b) (Null at 38.2°) Partial(WSR)	Fig. 5.4(c) (Null at 60.8°) Partial(WSR)
No. of Controlled Elements		4	4	4
SLV (dB)		2.0575	2.9552	2.3319
DIRECTIVITY	8	8.5754	8.6479	9.3738
HPBW (DEG.)	12.8025	12.0888	12.0572	12.1054
SLL (dB)	-12.8	-14.3703	-14.9354	-15.5587
Initial Null Depth(dB)		-12.8	-16.42	-17.89
Final Null Depth(dB)		-60.49	-62.84	-63.54
No. of Generations		279	566	540
CPU time (Sec)		1116	2264	2160

Table 5.8: Computed Array Parameters for Fig. 5.4

5.3 Simulation results of 8 element chebyshev partially adaptive arrays

5.3.1 Introduction

In this section, we study the array performance and behavior of its parameters such as half-power beam width (HPBW), directivity , sidelobe level (SLL), the minimum number of controlled elements K and the sidelobe variation (SLV) on 8-element chebyshev array [43] with sidelobe level of -30dB and with element spacing of 0.5 λ. The validity of the proposed partially adaptive method is examined by first placing a single null on the peak of each of the sidelobes separately and then placing two nulls on the peak of first and second sidelobes. The results are compared with the fully adaptive case with and without sidelobe restrictions.

5.3.2 Simulation results

The results of Fig.5.5 show one null which has been steered to the peak of the first sidelobe level at 26.6°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.5(a) when all the 8-elements are perturbed without restricting the sidelobe variation. Fig.5.5(b) shows the perturbed pattern compared to the initial, when all the 8-elements are perturbed and the sidelobe variation is restricted to 0.74 dB. Fig.5.5(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=2 in this case. The required null has been achieved precisely, but the sidelobe level variation has changed to more than 9 dB. Fig.5.5(d) shows the pattern when the sidelobe variation is restricted to 1.5 dB while achieving the required null. K is equal to 3 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.5 are given in table 5.9 and table 5.10 respectively.

The results of Fig.5.6 (a) and (b) show one null, which has been steered to the peak of the second sidelobe level at 40.2° and the results of Fig.5.6 (c) and (d) show one null, which has been steered to the peak of the third sidelobe level at 61.6°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.6(a) when all the 8-elements are perturbed and the sidelobe variation is restricted to 0.49 dB. Fig.5.6(b) shows the pattern when the sidelobe variation is restricted to 2.8 dB while achieving the required null. K is equal to 3 in this case. Fig.5.6(c) shows the

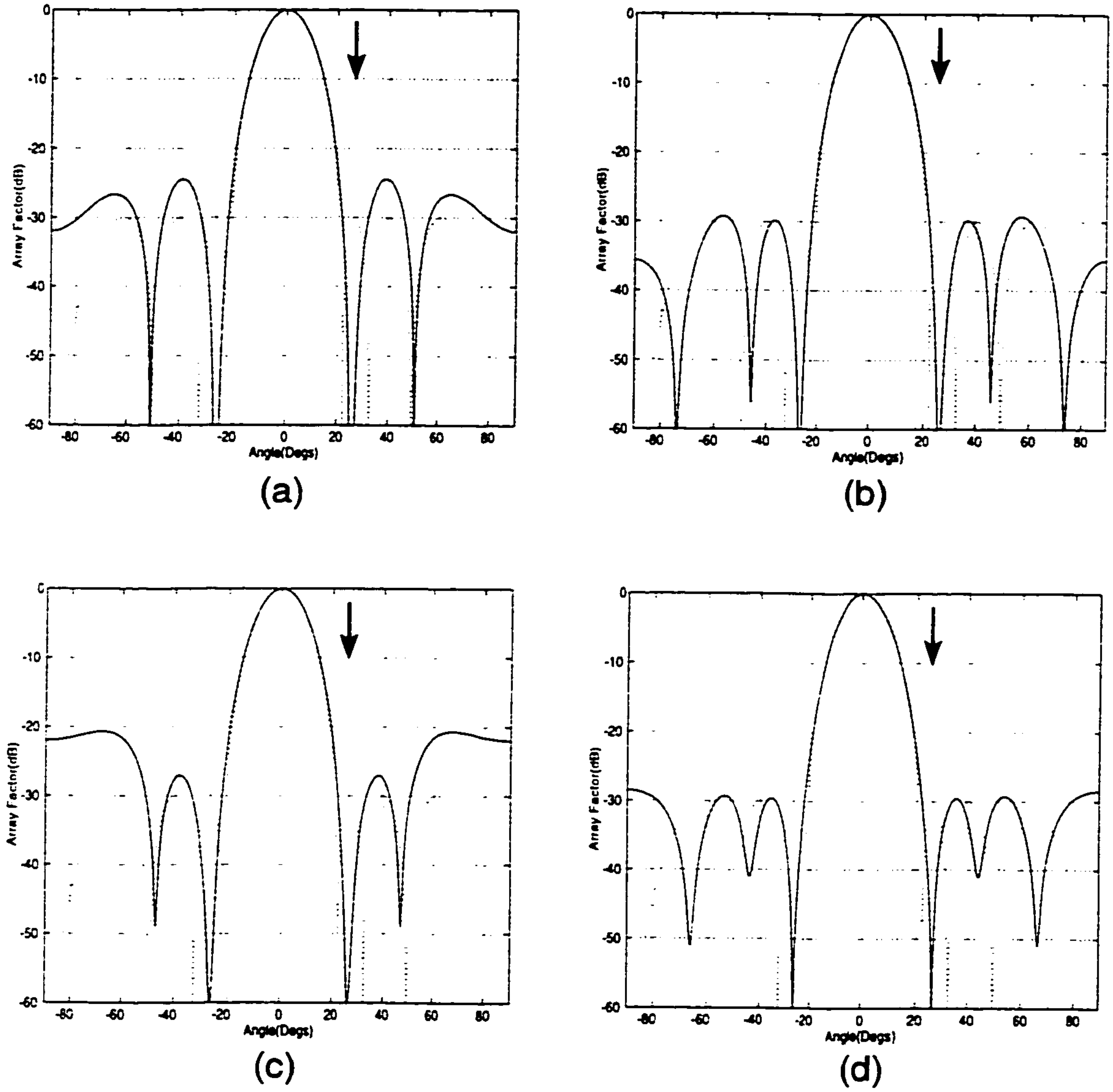


Figure 5.5: Array patterns of 8 element chebyshev array with -30 dB side lobe level and one null imposed on the peak of first sidelobe at 26.6° (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 0.74 dB (c)Minimum (optimum) number of elements are controlled (K=2) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=3) with sidelobes restricted to 1.5 dB

ELEMENT Number	Fig. 5.5(a) Full(WOSR) Δ_n	Fig. 5.5(b) Full(WSR) Δ_n	Fig. 5.5(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.5(d) Partial(WSR) $\hat{\Delta}_n$
1	-0.0035	-0.0522	0	-0.0192
2	-0.0100	-0.0006	0	0
3	0.0241	0.0140	0.0385	0
4	0.0134	0.0047	0	0
5	-0.0121	-0.0095	0	0
6	-0.0230	-0.0196	-0.0435	0
7	0.0121	-0.0008	0	0.0460
8	0.0094	0.0512	0	0.1926

Table 5.9: Computed element position perturbations for Fig. 5.5 when a single null is steered at 26.6°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.5(a) Full(WOSR)	Fig. 5.5(b) Full(WSR)	Fig. 5.5(c) Partial(WOSR)	Fig. 5.5(d) Partial(WSR)
Min. Controlled Elements K		8	8	2	3
SLV (dB)		5.5675	0.7419	9.3232	1.5071
DIRECTIVITY	6.7326	6.6524	6.7166	6.5543	6.8831
HPBW (DEG.)	16.4440	16.5309	16.4044	16.6766	16.0223
SLL (dB)	-30	-24.4325	-29.2581	-20.6768	-28.4929
Null Depth(dB)	-30	-65.36	-64.46	-60.00	-64.40
No. of Generations		31	70	52	201
CPU time (Sec)		10.33	280	17.33	804

Table 5.10: Computed Array Parameters for Fig. 5.5 when a single null is steered at 26.6°.

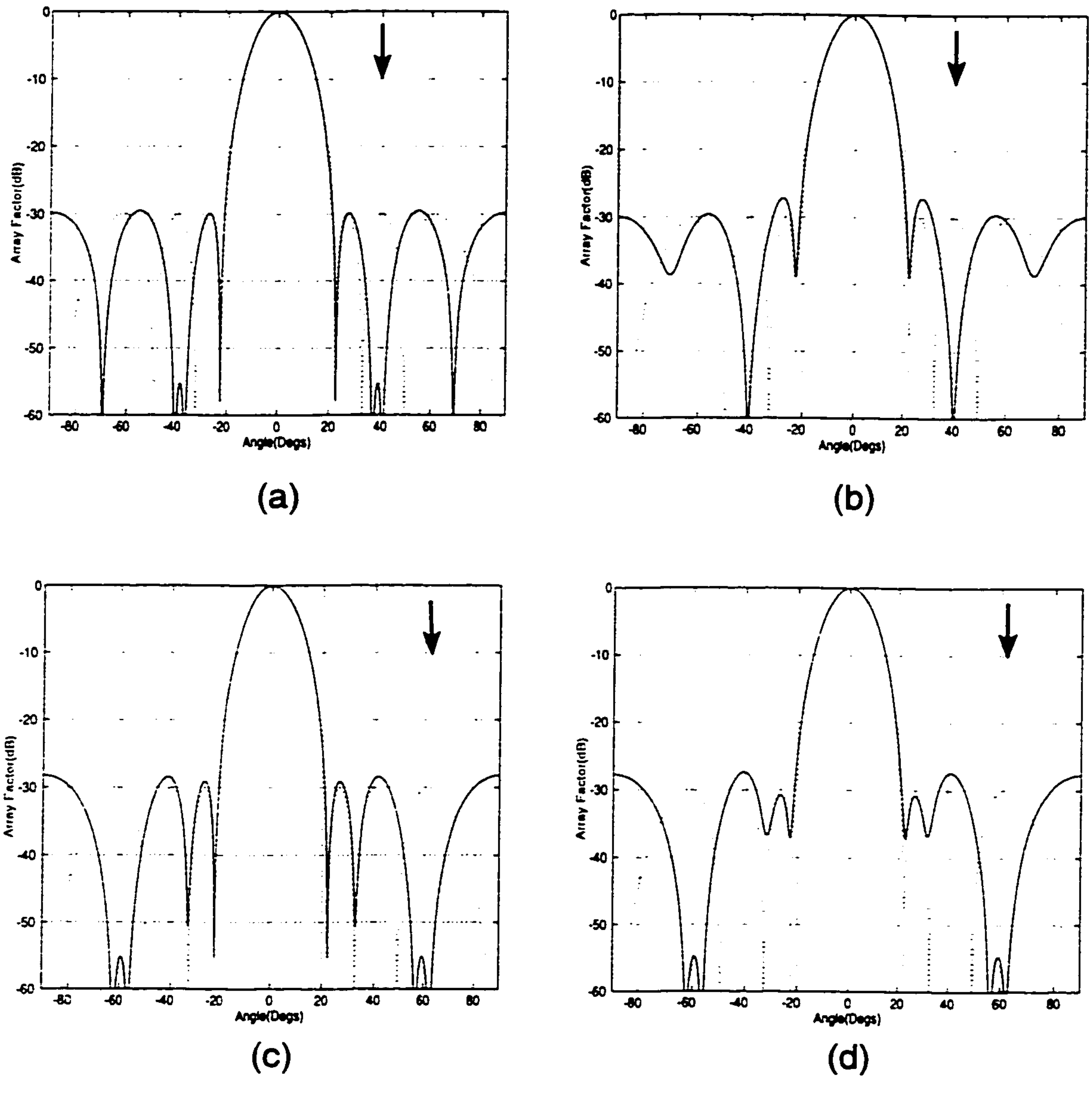


Figure 5.6: Array patterns of 8 element chebyshev array with (a) one null imposed on the peak of second sidelobe at 40.2° when all elements are controlled and sidelobes restricted to 0.49 dB (b) Minimum (optimum) number of elements are controlled (K=3) with sidelobes restricted to 2.8 dB (c) one null imposed on the peak of third sidelobe at 61.6° when all elements are controlled and sidelobes restricted to 1.77 dB (d) Minimum (optimum) number of elements are controlled (K=2) with sidelobes restricted to 2.58 dB

resulting pattern when all the 8-elements are perturbed and the sidelobe variation is restricted to 1.77 dB. Fig.5.6(d) shows the pattern when the sidelobe variation is restricted to 2.58 dB while achieving the required null. K is equal to 2 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.6 are given in table 5.11 and table 5.12 respectively.

In general for the case of a single null in 8-element chebyshev partially adaptive

ELEMENT Number	Fig. 5.6(a) (Null at 40.2°) Full(WSR) Δ_n	Fig. 5.6(b) (Null at 40.2°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.6(c) (Null at 61.6°) Full(WSR) Δ_n	Fig. 5.6(d) (Null at 61.6°) Partial(WSR) $\hat{\Delta}_n$
1	-0.0245	0	0.0065	0
2	0.0235	0.0507	0.0101	0
3	0.0003	0	-0.0045	0
4	-0.0049	-0.0087	0.0082	0.0253
5	0.0036	0	-0.0069	0
6	-0.0009	0	0.0109	0
7	-0.0212	-0.0171	-0.0037	-0.0165
8	0.0256	0	-0.0039	0

Table 5.11: Computed element position perturbations for Fig. 5.6 as a function of λ.

arrays with sidelobe restriction, the same behavior obtained in case of uniform arrays is noticed here. It is observed that as we steer a single null towards the main beam from the peak of third sidelobe to the peak of first sidelobe, the minimum no of controlled elements K and the SLV increase, while the other array parameters such as directivity, HPBW remain almost unchanged. This is due to the reason that steering of a null near the main beam requires a higher order of degrees of freedom.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.6(a) (Null at 40.2°) Full (WSR)	Fig. 5.6(b) (Null at 40.2°) Partial (WSR)	Fig. 5.6(c) (Null at 61.6°) Full (WSR)	Fig. 5.6(d) (Null at 61.6°) Partial (WSR)
Min. Controlled Elements K		8	3	8	2
SLV (dB)		0.4916	2.8023	1.7764	2.5863
DIRECTIVITY	6.7326	6.7114	6.6688	6.7240	6.6999
HPBW (DEG.)	16.4440	16.4799	16.6108	16.4702	16.5172
SLL (dB)	-30	-29.5084	-27.1977	-28.2238	-27.4137
Null Depth(dB)	-30	-60.00	-60.78	-61.14	-60.54
No. of Generations		42	41	51	25
CPU time (Sec)		168	164	204	100

Table 5.12: Computed Array Parameters for Fig. 5.6

It is also observed that, when the sidelobe restriction is applied, the number of controlled elements is increasing from the minimum possible value, which is due to increase in the number of constraints in this optimization. Tables 5.10 and 5.12, show a comparison of array parameters such as directivity and HPBW. The results show that they are almost unchanged. The sidelobe variation (SLV) and sidelobe level (SLL) in case of partially adaptive arrays is slightly higher than in fully adaptive case, since it is obvious that the best array parameters are obtained, when all the 8-elements are perturbed, as it affords the greatest control over the array response.

The results of Fig.5.7 show two nulls, which has been steered to the peaks of the first and second sidelobe levels at 26.6° and 40.2°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.7(a) when all the 8-elements are per-

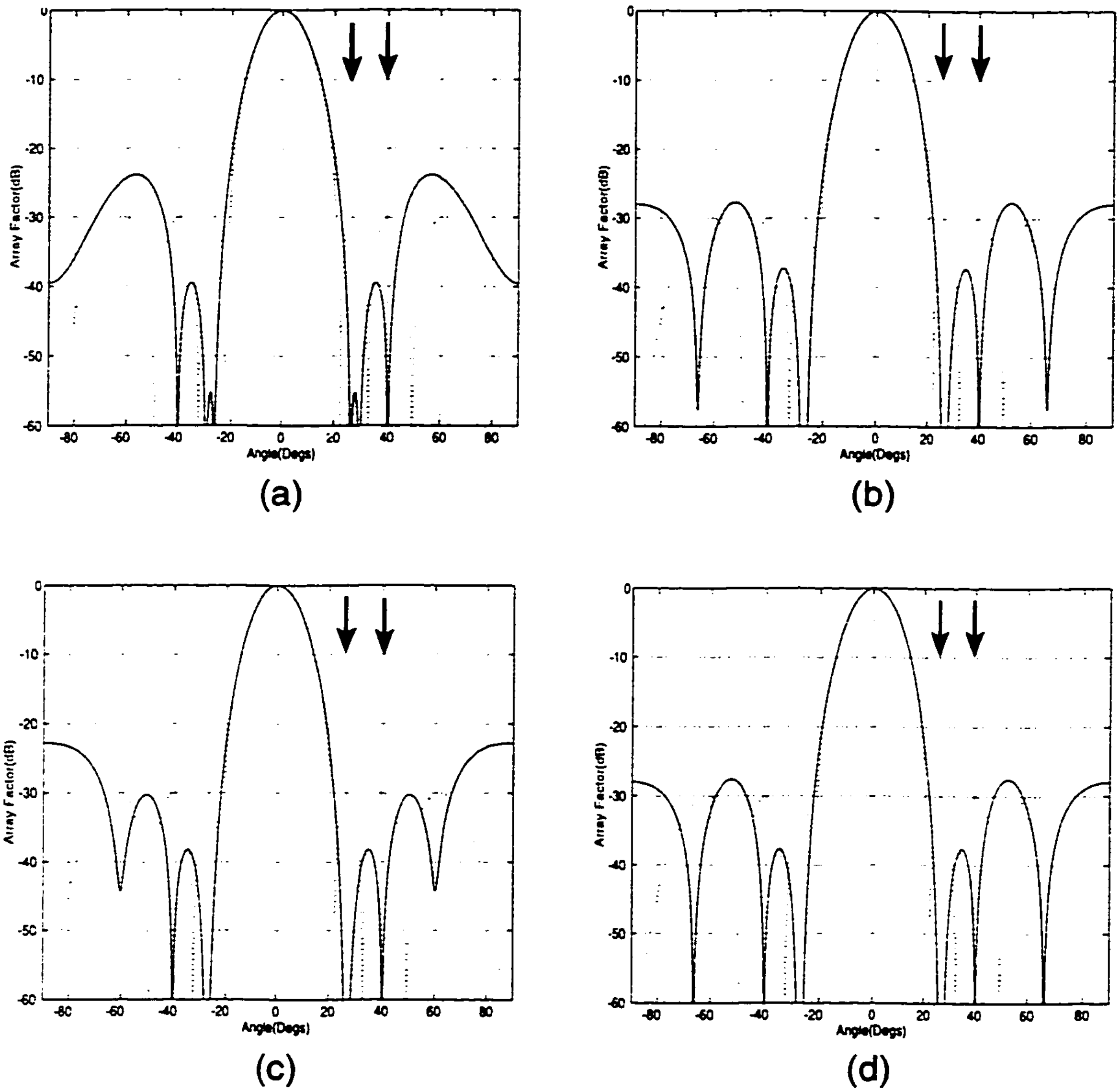


Figure 5.7: Array patterns of 8 element chebyshev array with -30 dB side lobe level and with two nulls imposed on the peak of first and second sidelobes at 26.6° and 40.2° (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 2.29 dB (c)Minimum (optimum) number of elements are controlled (K=4) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=5) with sidelobes restricted to 2.41 dB

turbed without restricting the sidelobe variation. Fig.5.7(b) shows the perturbed pattern compared to the initial, when all the 8-elements are perturbed and the side-lobe variation is restricted to 2.29 dB. Fig.5.7(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=4 in this case. The required nulls has been achieved precisely, but the sidelobe level variation has changed to more than 7 dB. Fig.5.7(d) shows the pattern when the sidelobe variation is restricted to 2.41 dB while achieving the required nulls. K is equal to 5 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.7 are given in table 5.13 and table 5.14 respectively.

For the case of two nulls in 8-element chebyshev partially adaptive array with side-

ELEMENT Number	Fig. 5.7(a) (Nulls at 26.6°,40.2°) Full(WOSR) Δ_n	Fig. 5.7(b) (Nulls at 26.6°,40.2°) Full(WSR) Δ_n	Fig. 5.7(c) (Nulls at 26.6°,40.2°) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.7(d) (Nulls at 26.6°,40.2°) Partial(WSR) $\hat{\Delta}_n$
1	-0.0451	-0.0874	-0.1177	-0.0956
2	0.0259	0.0001	0	0
3	0.0317	0.0089	0	0.0084
4	-0.0048	0.0010	0.0066	0
5	-0.0004	0.0019	-0.0018	0
6	-0.0369	-0.0040	0	-0.0086
7	-0.0315	0.0058	0	-0.0018
8	0.0379	0.1044	0.0943	0.0891

Table 5.13: Computed element position perturbations for Fig. 5.7 when two nulls are steered at 26.6° and 40.2°. Perturbations are given as a function of λ.

lobe restriction, the same behavior obtained in case of uniform arrays is noticed here. It is observed that as we increase the number of nulls from one to two the minimum

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.7(a) Full(WOSR)	Fig. 5.7(b) Full(WSR)	Fig. 5.7(c) Partial(WOSR)	Fig. 5.7(d) Partial(WSR)
Min. Controlled Elements K		8	8	4	5
SLV (dB)		6.1606	2.2980	7.2213	2.4128
DIRECTIVITY	6.7326	6.5881	6.7986	6.8082	6.7803
HPBW (DEG.)	16.4440	16.6621	16.1856	16.1473	16.2275
SLL (dB)	-30	-23.8394	-27.7020	-22.7787	-27.5872
Null Depth(dB)	-30	-65.80	-60.10	-63.43	-62.21
No. of Generations		189	2782	445	192
CPU time (Sec)		63	11128	148.3	768

Table 5.14: Computed Array Parameters for Fig. 5.7 when two nulls are steered at 26.6° and 40.2°.

no of controlled elements K, SLL and SLV has also increased. From the table 5.14 comparing the array parameters of partially adaptive array, with the corresponding fully adaptive array, when two nulls are imposed at first and second sidelobes, one notices that directivity and HPBW are almost unchanged. The sidelobe variation (SLV) and SLL in case of partially adaptive arrays is slightly higher than in fully adaptive case, since it is obvious that the best array parameters are obtained, when all the 8-elements are perturbed, as it affords the greatest control over the array response.

5.3.3 Determination of the realizable minimum number of controlled elements.

To implement this technique for a single null, the null location which requires the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steering. In the case of 8-element chebyshev partially adaptive arrays a null on the peak of first sidelobe requires the use of 3 controlled elements. Now keeping the location of these 3 controlled elements fixed, a single null is imposed on the peak of first sidelobe as shown in Fig.5.8(a) and a single null is imposed on the peak of third sidelobe as shown in Fig.5.8(c). Therefore perturbing only these 3 fixed elements out of 8 elements, we can steer a single null anywhere in the sidelobe region of an 8-element chebyshev partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.8 are given in table 5.15 and table 5.16 respectively.

The results given in table 5.16 show that the sidelobe variation (SLV) when the null is imposed at second or third sidelobe, is higher than the previous results inspite of increasing the no. of controlled elements. This shows that for a particular null location, the SLV depends on the location of the minimum controlled elements. The 8-element chebyshev partially adaptive array is not suitable for steering two or more nulls, because steering two nulls requires a minimum of 5 controlled elements out of

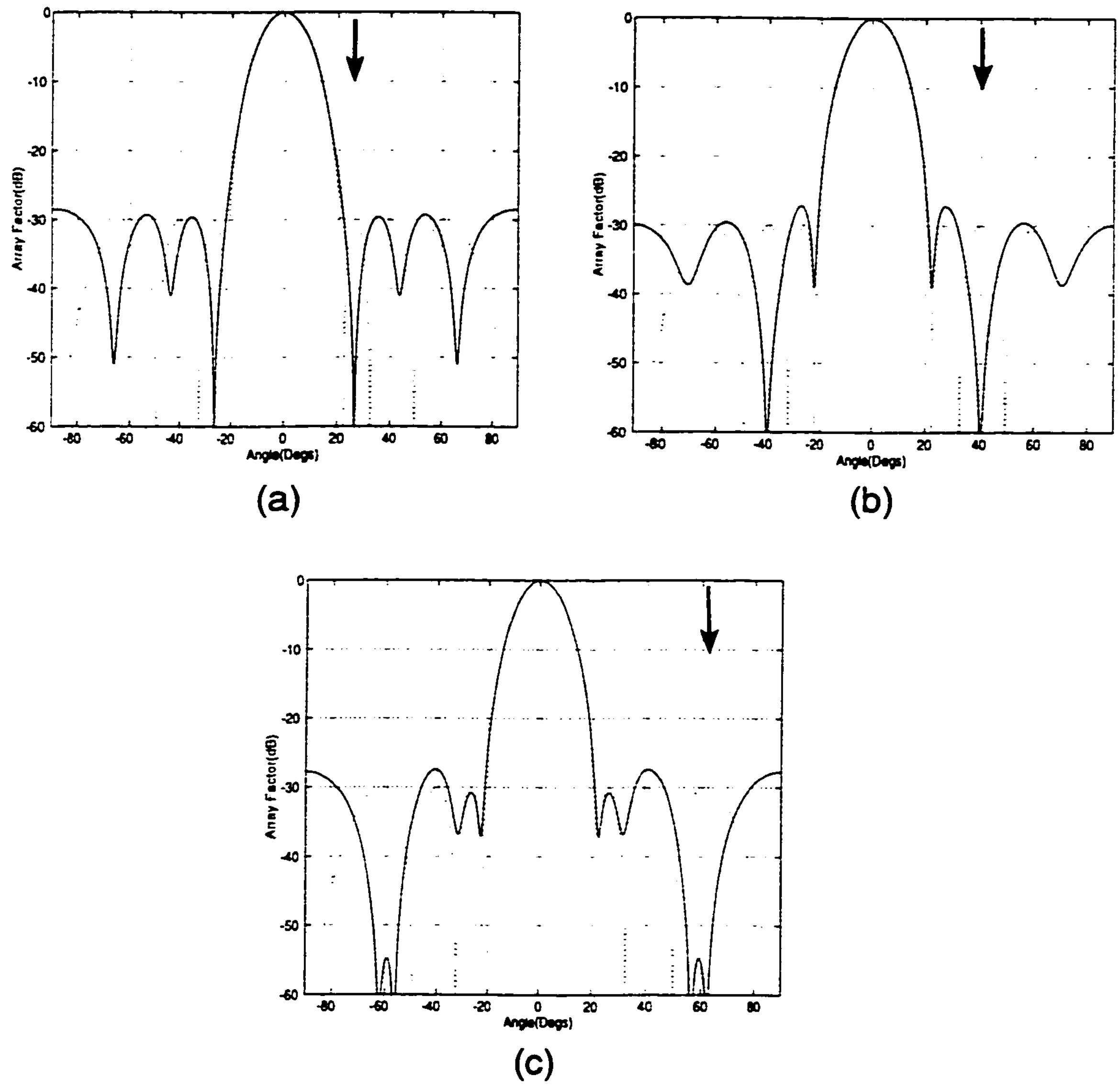


Figure 5.8: Array patterns of 8 element chebyshev array with -30 dB sidelobe level and perturbing 3 fixed elements with (a) one null imposed on the peak of first sidelobe at 26.6° and sidelobes restricted to 1.5 dB (b) one null imposed on the peak of second sidelobe at 40.2° and sidelobes restricted to 4.31 dB(c)one null imposed on the peak of third sidelobe at 61.6° and sidelobes restricted to 3.52 dB

ELEMENT Number	Fig. 5.8(a) (Null at 26.6°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.8(b) (Null at 40.2°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.8(c) (Null at 61.6°) Partial(WSR) $\hat{\Delta}_n$
1	-0.0192	0.0057	-0.0565
7	0.0460	-0.0674	0.0639
8	0.1926	0.0364	0.2440

Table 5.15: Computed element position perturbations for Fig. 5.8 as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.8(a) (Null at 26.6°) Partial(WSR)	Fig. 5.8(b) (Null at 40.2°) Partial(WSR)	Fig. 5.8(c) (Null at 61.6°) Partial(WSR)
No. of Controlled Elements		3	3	3
SLV (dB)		1.5071	4.3160	3.5256
DIRECTIVITY	6.7326	6.8831	6.6951	6.9327
HPBW (DEG.)	16.4440	16.0223	16.5741	15.8546
SLL (dB)	-30	-28.4929	-25.6840	-26.4744
Null Depth(dB)	-30	64.40	67.00	60.54
No. of Generations		588	541	526
CPU time (Sec)		2352	2164	2104

Table 5.16: Computed Array Parameters for Fig. 5.8

8 elements.

5.4 Simulation results of 16 element uniform partially adaptive arrays

5.4.1 Introduction

In this section, we study the array performance and behavior of its parameters such as half-power beam width (HPBW), directivity , sidelobe level (SLL), the minimum number of controlled elements K and the sidelobe variation (SLV) on 16-element uniform array with element spacing of 0.5 λ.

The validity of the proposed partially adaptive method is examined by first placing a single null and then placing up to four multiple nulls on the peaks of sidelobes. The results are compared with the fully adaptive case with and without sidelobe restrictions.

5.4.2 Simulation results

The results of Fig.5.9 show one null, which has been steered to the peak of the second sidelobe level at 17.9°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.9(a) when all the 16-elements are perturbed without restricting the sidelobe variation. Fig.5.9(b) shows the perturbed pattern compared

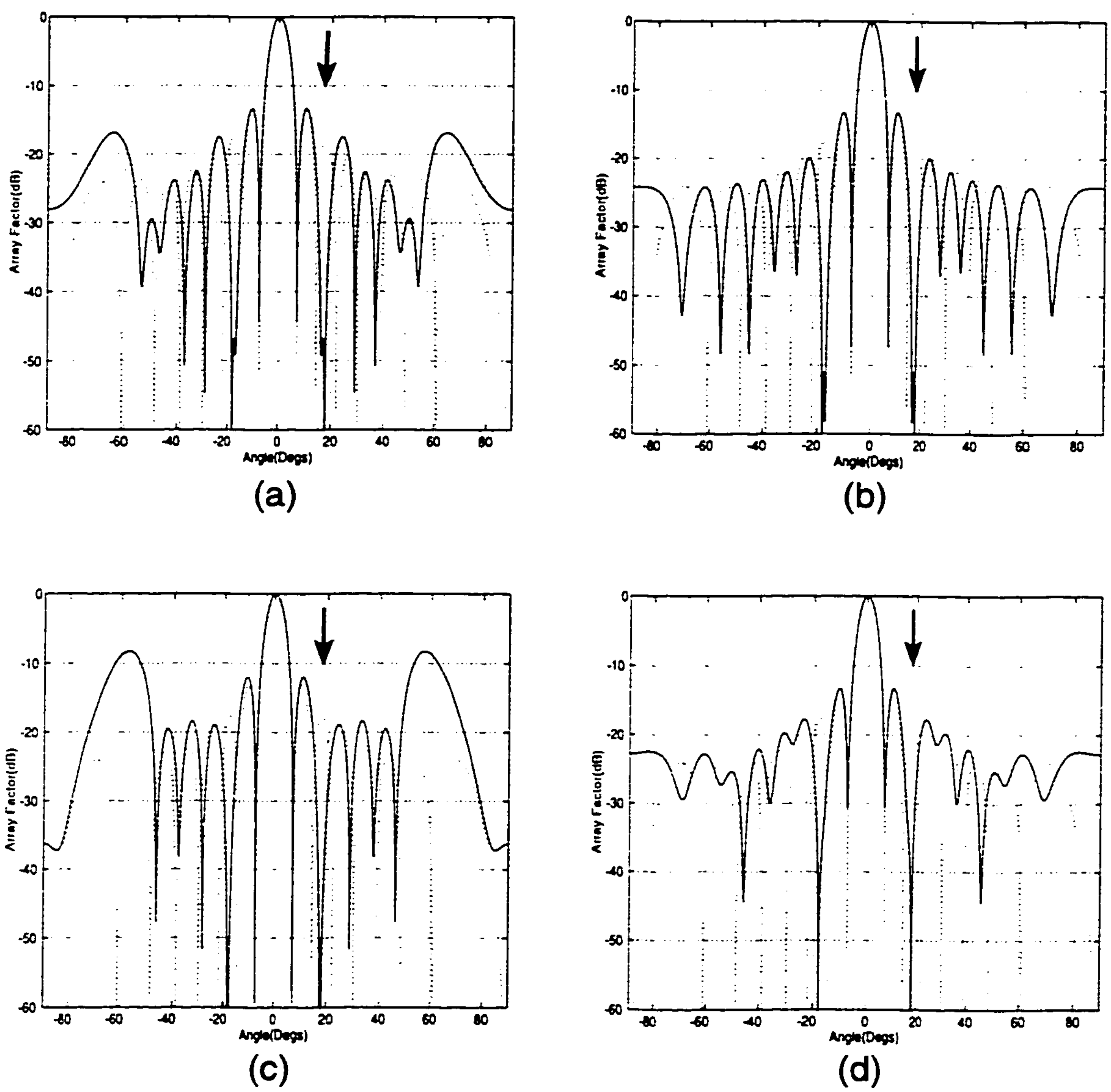


Figure 5.9: Array patterns of 16 element uniform array with one null imposed on the peak of second sidelobe at 17.9° (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 0.16 dB (c)Minimum (optimum) number of elements are controlled (K=4) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=7) with sidelobes restricted to 2.36 dB

to the initial, when all the 16-elements are perturbed and the sidelobe variation is restricted to 0.16 dB. Fig.5.9(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=4 in this case. The required null has been achieved precisely, but the sidelobe level variation has changed to more than 15 dB. Fig.5.9(d) shows the pattern when the sidelobe variation is restricted to 2.36 dB while achieving the required null. K is equal to 7 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.9 are given in table 5.17 and table 5.18 respectively.

The results of Fig.5.10 (a) and (b) show one null, which has been steered to the peak of the fourth sidelobe level at 34.1° and the results of Fig.5.10 (c) and (d) show one null, which has been steered to the peak of the sixth sidelobe level at 54.3°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.10(a) when all the 16-elements are perturbed and the sidelobe variation is restricted to 0.48 dB. Fig.5.10(b) shows the pattern when the sidelobe variation is restricted to 1.91 dB while achieving the required null. K is equal to 4 in this case. Fig.5.10(c) shows the resulting pattern when all the 16-elements are perturbed and the sidelobe variation is restricted to 0.46 dB. Fig.5.10(d) shows the pattern when the sidelobe variation is restricted to 1.47 dB while achieving the required null. K is equal to 4 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.10 are given in table 5.19 and table 5.20 respectively.

In general for the case of a single null in 16-element uniform partially adaptive

ELEMENT Number	Fig. 5.9(a) Full(WOSR) Δ_n	Fig. 5.9(b) Full(WSR) Δ_n	Fig. 5.9(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.9(d) Partial(WSR) $\hat{\Delta}_n$
1	-0.1267	-0.1638	0	-0.1481
2	0.0132	0.0096	0	0
3	0.1435	0.1278	0.2484	0.2493
4	0.1229	0.0710	0	0.2061
5	0.0745	0.0067	0	0
6	-0.1003	-0.0768	0	-0.1328
7	-0.1123	-0.1073	-0.3148	-0.2269
8	-0.1202	-0.0782	0	-0.1504
9	0.1161	0.0590	0	0
10	0.0891	0.1398	0.3050	0
11	0.0970	0.0753	0	0
12	-0.0591	-0.0300	0	0
13	-0.1250	-0.1187	0	0
14	-0.0847	-0.1375	-0.2936	0
15	-0.0183	-0.0346	0	0
16	0.1120	0.1500	0	0.2009

Table 5.17: Computed element position perturbations for Fig. 5.9 when a single null is steered at 17.9°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.9(a) Full(WOSR)	Fig. 5.9(b) Full(WSR)	Fig. 5.9(c) Partial(WOSR)	Fig. 5.9(d) Partial(WSR)
Min. Controlled Elements K		16	16	4	7
SLV (dB)		7.1760	0.1677	15.4507	2.3695
DIRECTIVITY	16	15.6975	16.2226	12.2096	16.7212
HPBW (DEG.)	6.3588	6.3746	6.3463	6.4213	6.3210
SLL (dB)	-13.22	-13.3255	-13.3315	-8.2693	-13.2846
Null Depth(dB)	-17.49	-68.36	-68.95	-76.03	-75.77
No. of Generations		147	199	359	274
CPU time (Sec)		49	796	119	1096

Table 5.18: Computed Array Parameters for Fig. 5.9 when a single null is steered at 17.9°.

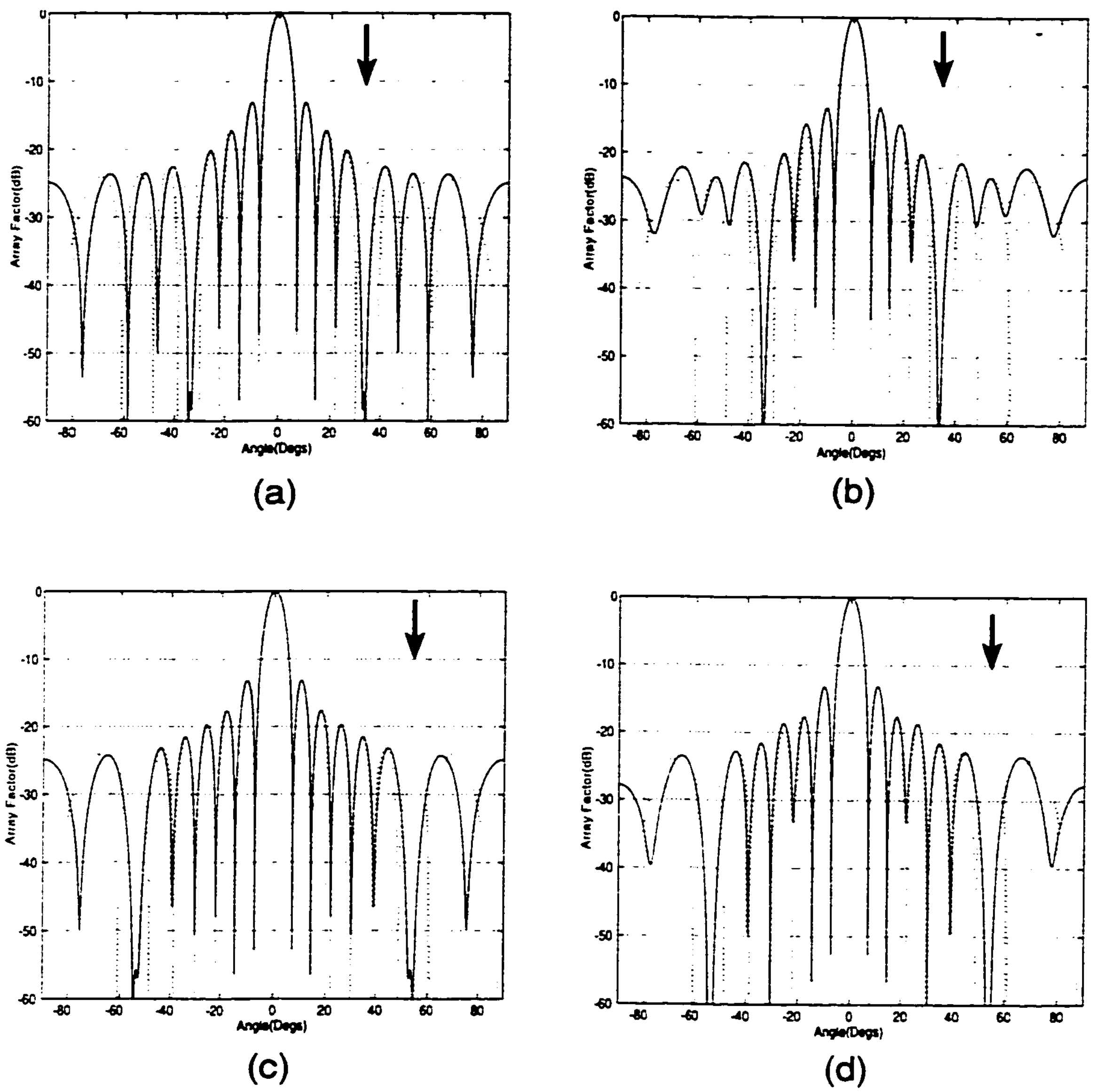


Figure 5.10: Array patterns of 16 element uniform array with (a) one null imposed on the peak of fourth sidelobe at 34.1° when all elements are controlled and sidelobes restricted to 0.48 dB (b) Minimum (optimum) number of elements are controlled (K=4) with sidelobes restricted to 1.91 dB (c) one null imposed on the peak of sixth sidelobe at 54.3° when all elements are controlled and sidelobes restricted to 0.46 dB (d) Minimum (optimum) number of elements are controlled (K=4) with sidelobes restricted to 1.47 dB

ELEMENT Number	Fig. 5.10(a) (Null at 34.1°) Full(WSR) Δ_n	Fig. 5.10(b) (Null at 34.1°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.10(c) (Null at 54.3°) Full(WSR) Δ_n	Fig. 5.10(d) (Null at 54.3°) Partial(WSR) $\hat{\Delta}_n$
1	-0.0324	0	-0.0060	0
2	0.0443	0.0946	0.0254	0
3	0.0019	0	-0.0301	-0.0664
4	-0.0401	0	0.0326	0
5	0.0004	0	-0.0108	0
6	0.0434	0.0904	-0.0002	0
7	-0.0181	0	0.0057	0
8	-0.0143	0	-0.0062	-0.0569
9	0.0359	0	0.0299	0
10	0.0293	0	-0.0069	0
11	-0.0446	-0.1015	-0.0004	0
12	0.0102	0	0.0110	0
13	0.0610	0.1123	-0.0291	-0.0606
14	0.0020	0	0.0288	0
15	-0.0466	0	-0.0203	-0.0338
16	0.0284	0	0.0004	0

Table 5.19: Computed element position perturbations for Fig. 5.10 as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.10(a) (Null at 34.1°) Full (WSR)	Fig. 5.10(b) (Null at 34.1°) Partial (WSR)	Fig. 5.10(c) (Null at 54.3°) Full (WSR)	Fig. 5.10(d) (Null at 54.3°) Partial (WSR)
Min. Controlled Elements K		16	4	16	4
SLV (dB)		0.4822	1.9170	0.4636	1.4722
DIRECTIVITY	16	16.0341	15.9614	16.0158	15.9889
HPBW (DEG.)	6.3588	6.3496	6.3803	6.3634	6.3620
SLL (dB)	-13.22	-13.0987	-13.3751	-13.1394	-13.1493
Initial Null Depth(dB)		-21.83	-21.83	-23.72	-23.72
Final Null Depth(dB)		-63.39	-62.05	-60.00	-65.53
No. of Generations		89	117	49	97
CPU time (Sec)		356	468	196	388

Table 5.20: Computed Array Parameters for Fig. 5.10

arrays with sidelobe restriction, the same behavior obtained in case of 8-element arrays is noticed here. It is observed that as we steer a single null towards the main beam, the minimum no of controlled elements K and the SLV is increase. The reason for this behavior is that as we are steering a null from a lower energy concentrated area to a higher energy concentrated area, it requires a higher order of degrees of freedom.

It is also observed that, when the sidelobe restriction is applied, the number of controlled elements increase from the minimum possible value. This is due to increasing the number of constraints in this optimization problem. Tables 5.18, and 5.20, show a comparison of array parameters such as directivity, HPBW and SLL. The results show that they are almost unchanged. The sidelobe variation (SLV) in case of partially adaptive arrays is slightly higher than in fully adaptive case, since it is obvious that the best array parameters are obtained, when all the 16-elements are perturbed, as it affords the greatest control over the array response.

The results of Fig.5.11 (a) and (b) show two nulls, which has been steered to the peaks of the second and fourth sidelobe levels at 17.9° and 34.1° respectively. The results of Fig.5.11 (c) and (d) show three nulls, which has been steered to the peaks of the second, fifth and seventh sidelobe levels at 17.9°, 43.3° and 69.6° respectively. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.11(a) when all the 16-elements are perturbed and the sidelobe variation is restricted to 1.51 dB. Fig.5.11(b) shows the pattern when the sidelobe variation is restricted to 2.52 dB while achieving the required nulls. K is equal to 8 in this case. Fig.5.11(c) shows the resulting pattern when all the 16-elements are perturbed and the sidelobe variation is restricted to 3.47 dB. Fig.5.11(d) shows the pattern when the sidelobe variation is restricted to 5.06 dB while achieving the required nulls. K is equal to 11 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.11 are given in table 5.21 and table 5.22 respectively.

The results of Fig.5.12 show four nulls, which has been steered to the peaks of the second, fourth, sixth and seventh sidelobe levels at 17.9°, 34.1°, 54.3° and 69.6° respectively. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.12(a) when all the 16-elements are perturbed without restricting the sidelobe variation. Fig.5.12(b) shows the perturbed pattern compared to the initial, when all the 16-elements are perturbed and the sidelobe variation is restricted

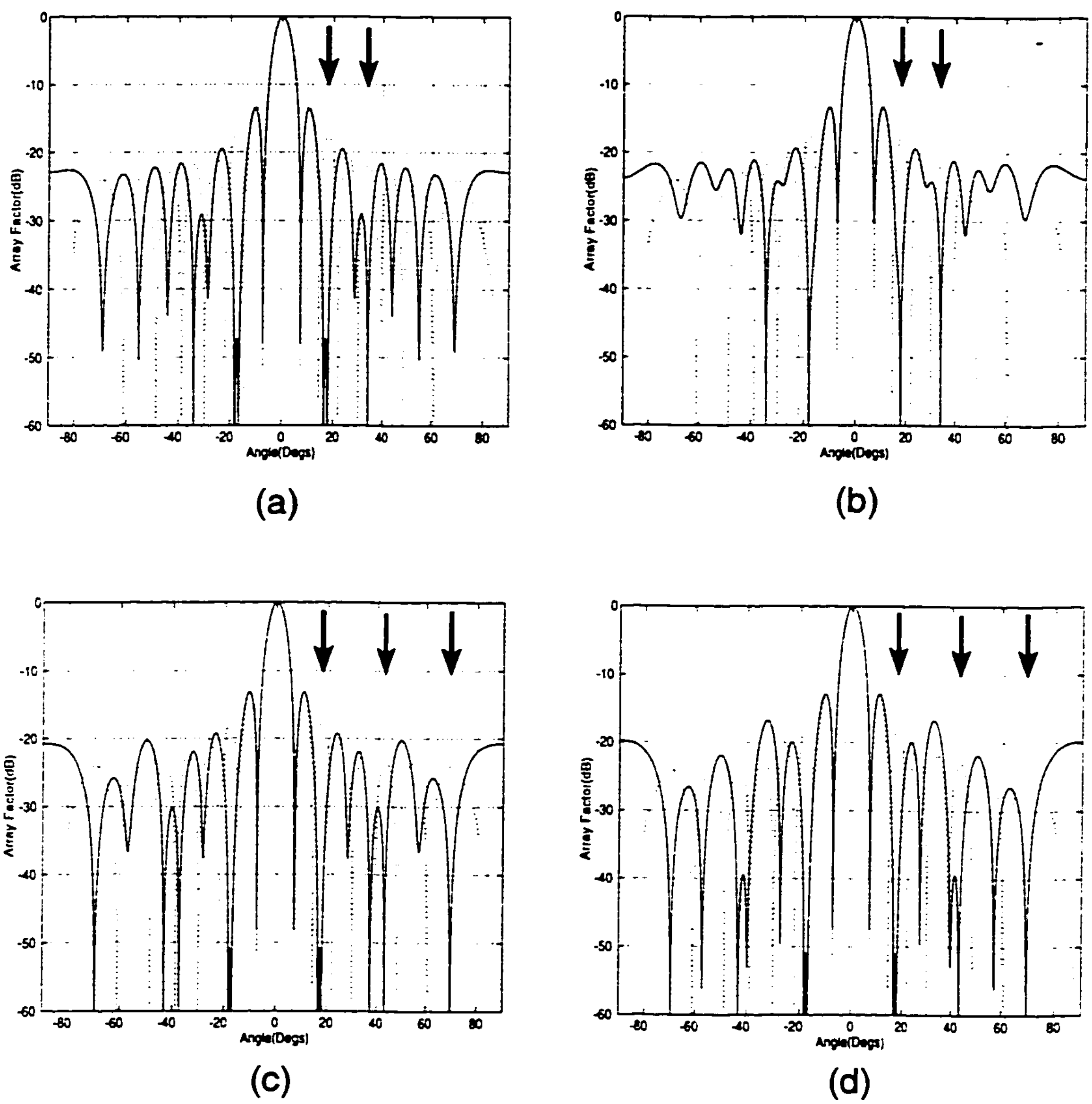


Figure 5.11: Array patterns of 16 element uniform array with (a) two nulls imposed on the peaks of second and fourth sidelobes at 17.9° and 34.1° respectively, when all elements are controlled and sidelobes restricted to 1.51 dB (b) Minimum (optimum) number of elements are controlled (K=8) with sidelobes restricted to 2.52 dB (c) three nulls imposed on the peaks of second, fifth and seventh sidelobes at 17.9°, 43.3° and 69.6° respectively, when all elements are controlled and sidelobes restricted to 3.47 dB (d) Minimum (optimum) number of elements are controlled (K=11) with sidelobes restricted to 5.06 dB

ELEMENT Number	Fig. 5.11(a) (Nulls at 17.9°, 34.1°) Full(WSR) Δ_n	Fig. 5.11(b) (Nulls at 17.9°, 34.1°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.11(c) (Nulls at 17.9°, 43.3°, 69.6°) Full(WSR) Δ_n	Fig. 5.11(d) (Nulls at 17.9°, 43.3°, 69.6°) Partial(WSR) $\hat{\Delta}_n$
1	-0.1781	-0.2484	-0.1343	-0.1382
2	0.0424	0	0.0507	0
3	0.1155	0	0.1237	0.1120
4	0.0690	0	0.0996	0.1234
5	0.0427	0	0.0534	0
6	-0.0399	0	-0.0844	-0.1460
7	-0.1298	0	-0.1273	-0.1557
8	-0.0997	0	-0.0490	-0.0045
9	0.0826	0.2112	0.0442	0
10	0.1333	0.2307	0.1761	0.1402
11	0.0518	0.1146	0.0812	0.1358
12	-0.0332	0	-0.0164	0
13	-0.0807	-0.1056	-0.1122	-0.1211
14	-0.1113	-0.1941	-0.1081	-0.1085
15	-0.0162	-0.0062	-0.0209	0
16	0.1891	0.2006	0.1381	0.1299

Table 5.21: Computed element position perturbations for Fig. 5.11 as a function of λ.

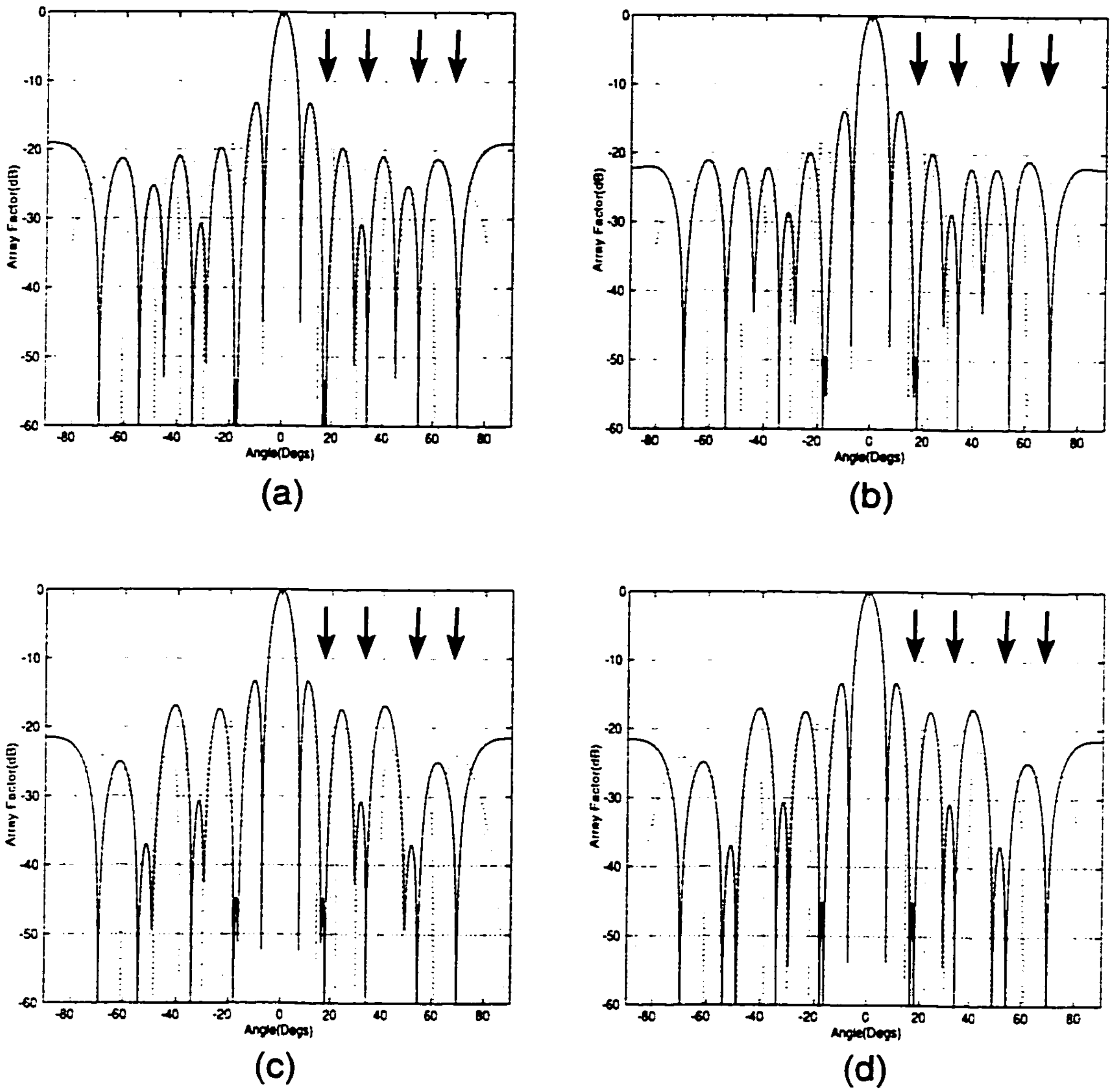


Figure 5.12: Array patterns of 16 element uniform array with four nulls imposed at 17.9°, 34.1°, 54.3° and 69.6° (peak of 2^{nd}, 4^{th}, 6^{th} and 7^{th} sidelobes). (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 3.02 dB (c)Minimum (optimum) number of elements are controlled (K=11) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=12) with sidelobes restricted to 5.97 dB

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.11(a) (Nulls at 17.9°, 34.1°) Full(WSR)	Fig. 5.11(b) (Nulls at 17.9°, 34.1°) Partial(WSR)	Fig. 5.11(c) (Nulls at 17.9°, 43.3°,69.6°) Full(WSR)	Fig. 5.11(d) (Nulls at 17.9°, 43.3°,69.6°) Partial(WSR)
Min. Controlled Elements K		16	8	16	11
SLV (dB)		1.5196	2.5252	3.4701	5.0665
DIRECTIVITY	16	16.2345	16.7145	16.0413	15.8299
HPBW (DEG.)	6.3588	6.3311	6.2684	6.3603	6.3268
SLL (dB)	-13.22	-13.5283	-13.3151	-13.1205	-12.8631
Null Depth(dB)	-17.49	-62.06	-65.22	-61.61	-60.09
No. of Generations		319	508	324	303
CPU time (Sec)		1276	2032	1296	1212

Table 5.22: Computed Array Parameters for Fig. 5.11

to 3.02 dB. Fig.5.12(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=11 in this case. The required null has been achieved precisely, but the sidelobe level variation has changed to more than 6 dB. Fig.5.12(d) shows the pattern when the sidelobe variation is restricted to 5.97 dB while achieving the required null. K is equal to 12 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.12 are given in table 5.23 and table 5.24 respectively.

For the case of multiple nulls in 16-element uniform partially arrays with sidelobe restriction, the same behavior obtained in case of 8-element arrays is noticed here. It is observed that, as we increase the number of nulls from one to four, the minimum no of controlled elements K, SLV and SLL has also increased. From the tables 5.22 and 5.24 comparing the array parameters of partially adaptive array,

ELEMENT Number	Fig. 5.12(a) Full(WOSR) Δ_n	Fig. 5.12(b) Full(WSR) Δ_n	Fig. 5.12(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.12(d) Partial(WSR) $\hat{\Delta}_n$
1	-0.1602	-0.1737	-0.1362	-0.1389
2	0.0545	0.0594	0	0
3	0.1112	0.1278	0.1026	0.1103
4	0.0912	0.0850	0.0848	0.1043
5	0.0065	0.0544	0	0
6	-0.0449	-0.0403	0	-0.0019
7	-0.1483	-0.1148	-0.1344	-0.1411
8	-0.1084	-0.0999	-0.1548	-0.1415
9	0.1073	0.0655	0.1458	0.1579
10	0.1315	0.1020	0.1526	0.1465
11	0.0311	0.0409	0.0059	0.0093
12	-0.0221	-0.0554	0	0
13	-0.0958	-0.1037	-0.1174	-0.0968
14	-0.1245	-0.1278	-0.1249	-0.1129
15	-0.0739	-0.0420	0	0
16	0.1361	0.1944	0.1473	0.1468

Table 5.23: Computed element position perturbations for Fig. 5.12 when four nulls are steered at 17.9°,34.1°,54.3° and 69.6°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.12(a) Full(WOSR)	Fig. 5.12(b) Full(WSR)	Fig. 5.12(c) Partial(WOSR)	Fig. 5.12(d) Partial(WSR)
Min. Controlled Elements K		16	16	11	12
SLV (dB)		5.0067	2.9682	6.1211	5.9799
DIRECTIVITY	16	16.0094	16.1944	15.8052	15.8168
HPBW (DEG.)	6.3588	6.3666	6.3642	6.3407	6.3385
SLL (dB)	-13.22	-13.1393	-13.8096	-13.3220	-13.3296
Null Depth(dB)	-17.49	-61.91	-60.53	-60.09	-60.01
No. of Generations		212	633	563	1023
CPU time (Sec)		70.6	2532	187	4092

Table 5.24: Computed Array Parameters for Fig. 5.12 when four nulls are steered at 17.9°,34.1°,54.3° and 69.6°.

with the corresponding fully adaptive array, when multiple nulls are imposed, one notices that directivity, HPBW, SLL are almost unchanged. The sidelobe variation (SLV) in case of partially adaptive arrays is slightly higher than in fully adaptive case, since it is obvious that the best array parameters are obtained, when all the 16-elements are perturbed, as it affords the greatest control over the array response.

5.4.3 Determination of the realizable minimum number of controlled elements

From the results obtained, it is found that the minimum number K and the location of the controlled elements in partially adaptive arrays depend on the number and location of nulls. In other words , the minimum number K and location of the controlled elements for partially adaptive method vary with the number and location of imposed nulls. This is not desirable in element position perturbation technique, since it requires to install a motor for each element and hence the number of motors are not being reduced. Therefore, this technique can only be realistically implemented if a limited number of motors are fixed at some optimum elements locations of an array, capable of providing a single or multiple imposed nulls at locations covering most of the sidelobe region, while other array parameters are nearly kept unchanged.

To implement this technique for a single null, the null location which requires

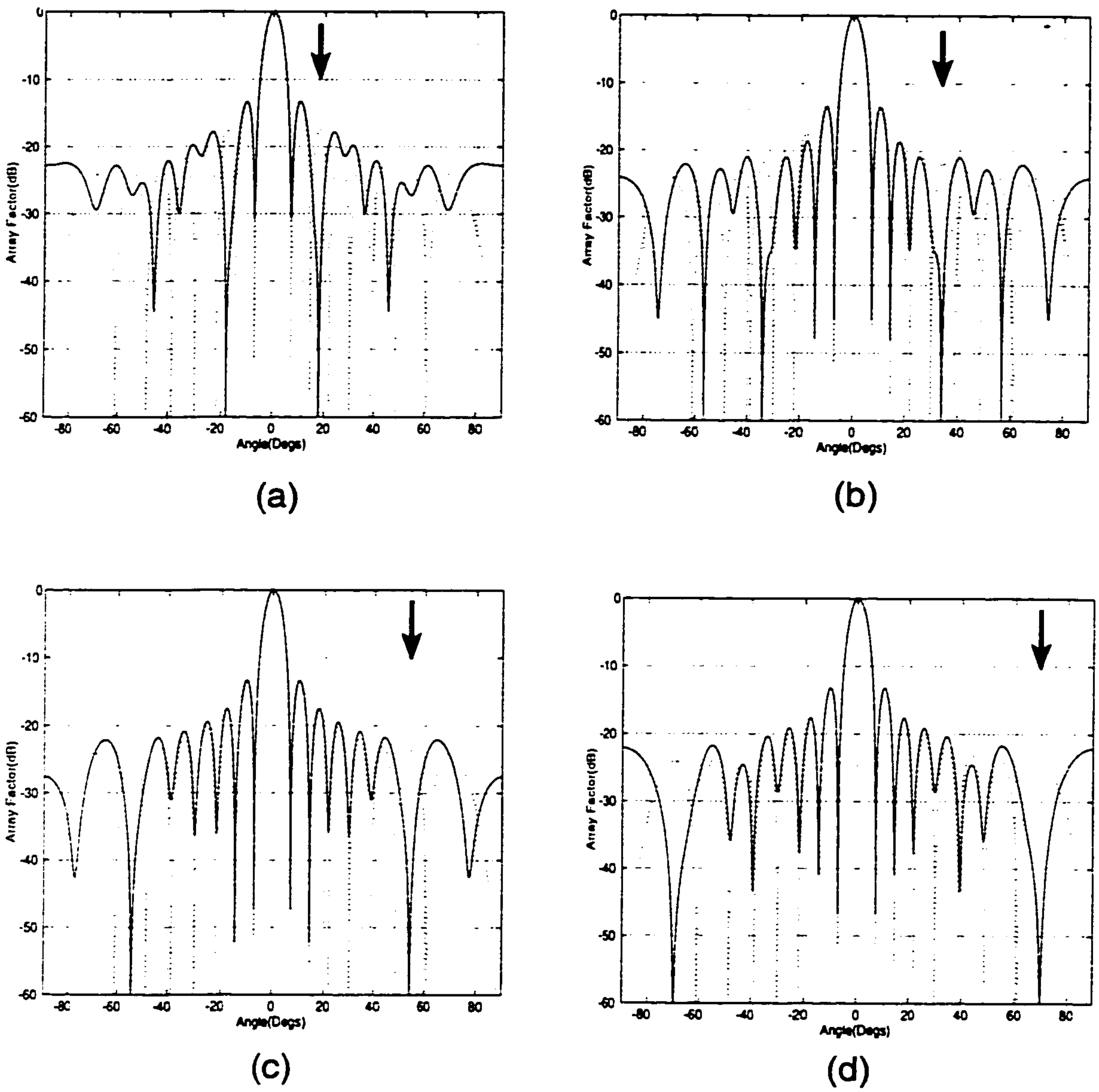


Figure 5.13: Array patterns of 16 element uniform array perturbing 7 fixed elements with (a) one null imposed on the peak of second sidelobe at 17.9° (b) one null imposed on the peak of fourth sidelobe at 34.1° (c) one null imposed on the peak of sixth sidelobe at 54.3° (d) one null imposed on the peak of seventh sidelobe at 69.6°

the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steering. In the case of 16-element uniform partially adaptive arrays, a null on the peak of second sidelobe requires the use of 7 controlled elements. Now keeping the location of these 7 controlled elements fixed, a single null is imposed on the peak of fourth sidelobe as shown in Fig.5.13(b) with sidelobes restricted to 1.99 dB. A single null is imposed on the peak of sixth sidelobe as shown in Fig.5.13(c) with sidelobes restricted to 1.96 dB and a single null is imposed on the peak of seventh sidelobe as shown in Fig.5.13(c) with sidelobes restricted to 1.98 dB. Therefore perturbing only these 7 fixed elements out of 16-elements. we can steer a single null any where in the sidelobe region of an 16-element uniform partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.13 are given in table 5.25 and table 5.26 respectively.

The results given in table 5.26 show that the sidelobe variation (SLV) when the null is imposed at fourth, sixth or seventh sidelobe, is higher than the previous results inspite of increasing the no. of controlled elements. This shows that for a particular null location, the SLV depends on the location of the minimum controlled elements.

To implement this technique for two nulls, the null locations which requires the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steering. In the case of 16-element uniform partially adaptive arrays, two nulls on the

ELEMENT Number	Fig. 5.13(a) (Null at 17.9°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.13(b) (Null at 34.1°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.13(c) (Null at 54.3°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.13(d) (Null at 69.6°) Partial(WSR) $\hat{\Delta}_n$
1	-0.1481	-0.1171	-0.0173	0.0454
3	0.2493	-0.0009	-0.0892	-0.0505
4	0.2061	-0.0892	0.0448	0.0337
6	-0.1328	0.0535	-0.0072	0.0224
7	-0.2269	-0.0491	0.0358	-0.0501
8	-0.1504	-0.0618	-0.0356	0.0378
16	0.2009	0.0991	0.0378	0.0536

Table 5.25: Computed element position perturbations for Fig. 5.13 as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig.5.13(a) (Null at 17.9°) Partial (WSR)	Fig.5.13(b) (Null at 34.1°) Partial (WSR)	Fig.5.13(c) (Null at 54.3°) Partial (WSR)	Fig.5.13(d) (Null at 69.6°) Partial (WSR)
No. of Controlled Elements		7	7	7	7
SLV (dB)		2.3695	1.9935	1.9612	1.9859
DIRECTIVITY	16	16.7212	16.4209	16.2046	16.1907
HPBW (DEG.)	6.3588	6.3210	6.2886	6.3346	6.3517
SLL (dB)	-13.22	-13.2846	-13.5455	-13.3819	-13.2044
Initial Null Depth(dB)		-17.49	-21.83	-23.72	-24.05
Final Null Depth(dB)		-75.77	-65.36	-65.53	-61.48
No. of Generations		274	148	108	79
CPU time (Sec)		1096	592	432	316

Table 5.26: Computed Array Parameters for Fig. 5.13

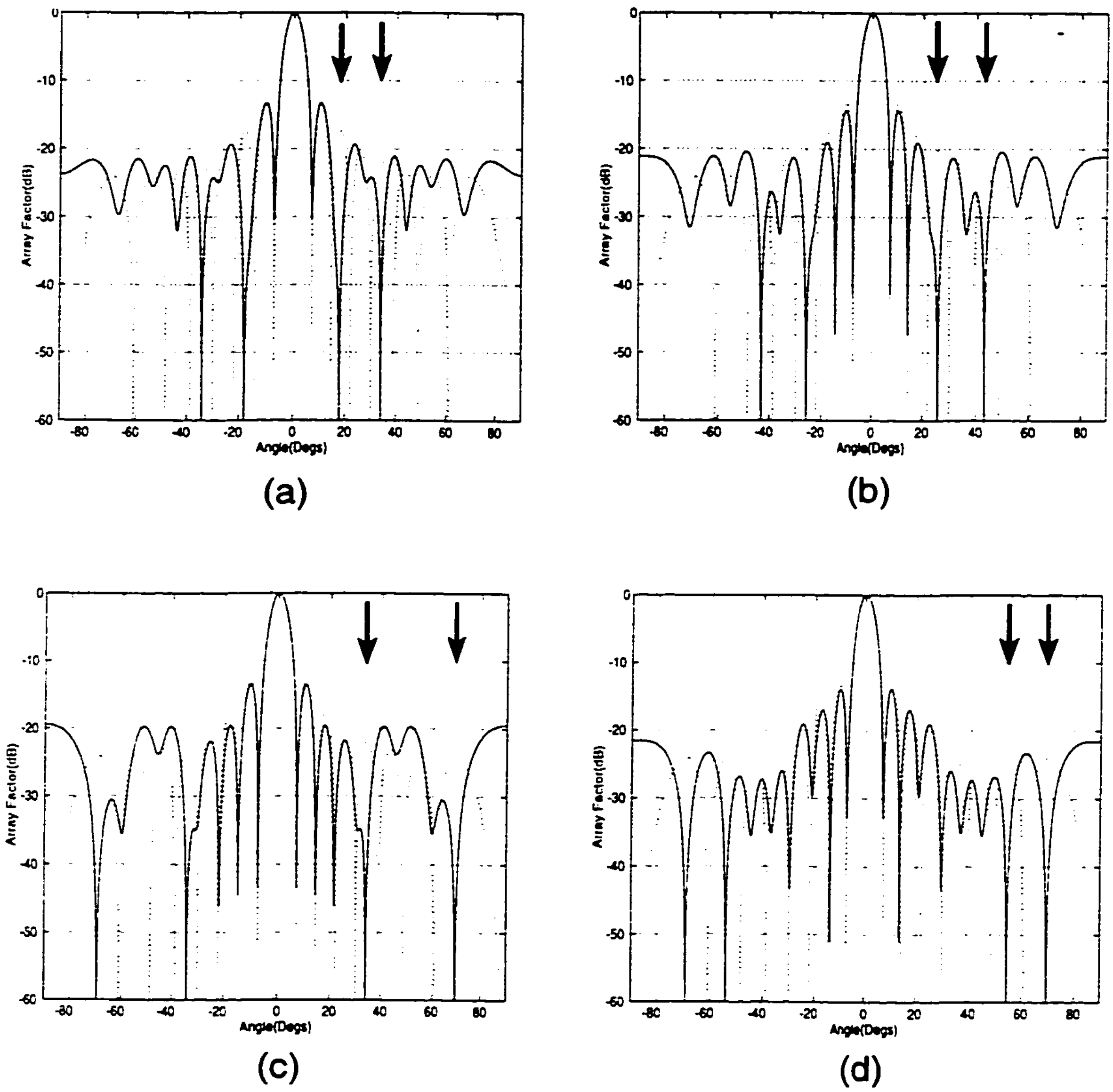


Figure 5.14: Array patterns of 16 element uniform array perturbing 8 fixed elements with (a) two nulls imposed on the peak of second and fourth sidelobes at 17.9° and 34.1° (b) two nulls imposed on the peak of third and fifth sidelobes at 25.8° and 43.3° (c) two nulls imposed on the peak of fourth and seventh sidelobes at 34.1° and 69.6° (d) two nulls imposed on the peak of sixth and seventh sidelobes at 54.3° and 69.6°

peaks of second and fourth sidelobes requires the use of 8 controlled elements. Now keeping the location of these 8 controlled elements fixed, two nulls are imposed on the peaks of third and fifth sidelobes as shown in Fig.5.14(b) with sidelobes restricted to 3.31 dB . Two nulls are imposed on the peaks of fourth and seventh sidelobes as shown in Fig.5.14(c) with sidelobes restricted to 4.58 dB and two nulls are imposed on the peaks of sixth and seventh sidelobe as shown in Fig.5.14(c) with sidelobes restricted to 2.49 dB. Therefore perturbing only these 8 fixed elements out of 16-elements, we can steer two nulls any where in the sidelobe region of an 16-element uniform partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.14 are given in table 5.27 and table 5.28 respectively.

The 16-element uniform partially adaptive array is not suitable for steering three

ELEMENT Number	Fig. 5.14(a) (Nulls at 17.9°,34.1°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.14(b) (Nulls at 25.8°,43.3°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.14(c) (Nulls at 34.1°,69.6°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.14(d) (Nulls at 54.3°,69.6°) Partial(WSR) $\hat{\Delta}_n$
1	-0.2484	-0.1339	-0.1003	-0.0705
9	0.2112	-0.0620	0.0283	0.0316
10	0.2307	-0.1474	0.1093	0.0113
11	0.1146	-0.0224	-0.0971	-0.0107
13	-0.1056	-0.0075	0.0444	0.1296
14	-0.1941	-0.1052	-0.0680	0.2455
15	-0.0062	-0.0728	-0.0384	0.1699
16	0.2006	0.1783	0.0893	0.2616

Table 5.27: Computed element position perturbations for Fig. 5.14 as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.14(a) (Nulls at 17.9°, 34.1°) Partial (WSR)	Fig. 5.14(b) (Nulls at 25.8°, 43.3°) Partial (WSR)	Fig. 5.14(c) (Nulls at 34.1°, 69.6°) Partial (WSR)	Fig. 5.14(d) (Nulls at 54.3°, 69.6°) Partial (WSR)
No. of Controlled Elements		8	8	8	8
SLV (dB)		2.5252	3.3139	4.5872	2.4931
DIRECTIVITY	16	16.7145	16.5268	16.4186	16.9894
HPBW (DEG.)	6.3588	6.2684	6.3325	6.3278	6.1612
SLL (dB)	-13.22	-13.3151	-14.3530	-13.5200	-13.8883
Initial Null Depth(dB)		-17.49	-20.14	-21.83	-23.72
Final Null Depth(dB)		-65.22	-61.25	-61.70	-60.01
No. of Generations		508	429	538	731
CPU time (Sec)		2032	1716	2152	2924

Table 5.28: Computed Array Parameters for Fig. 5.14

or more nulls, because steering three nulls requires a minimum of 11 controlled elements out of 16 elements.

5.5 Simulation results of 16 element chebyshev partially adaptive arrays

5.5.1 Introduction

In this section, we study the array performance and behavior of its parameters such as half-power beam width (HPBW), directivity , sidelobe level (SLL), the minimum number of controlled elements K and the sidelobe variation (SLV) on 16-element chebyshev array with sidelobe level of -30dB and with element spacing of 0.5 λ. The validity of the proposed partially adaptive method is examined by first placing a single null and then placing multiple nulls on the peak of sidelobes. The results are compared with the fully adaptive case with and without sidelobe restrictions.

5.5.2 Simulation results

The results of Fig.5.15 show one null, which has been steered to the peak of the second sidelobe level at 18.4°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.15(a) when all the 16-elements are perturbed without restricting the sidelobe variation. Fig.5.15(b) shows the perturbed pattern compared to the initial, when all the 16-elements are perturbed and the sidelobe variation is restricted to 0.38 dB. Fig.5.15(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=4 in this case. The required null has been achieved precisely, but the sidelobe level variation has changed to more than 9 dB. Fig.5.15(d) shows the pattern when the sidelobe variation is restricted to 3.48 dB while achieving the required null. K is equal to 7 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.15 are given in table 5.29 and table 5.30 respectively.

The results of Fig.5.16 (a) and (b) show one null, which has been steered to the peak of the fourth sidelobe level at 33.7° and the results of Fig.5.16 (c) and (d) show one null, which has been steered to the peak of the sixth sidelobe level at 53.9°. The

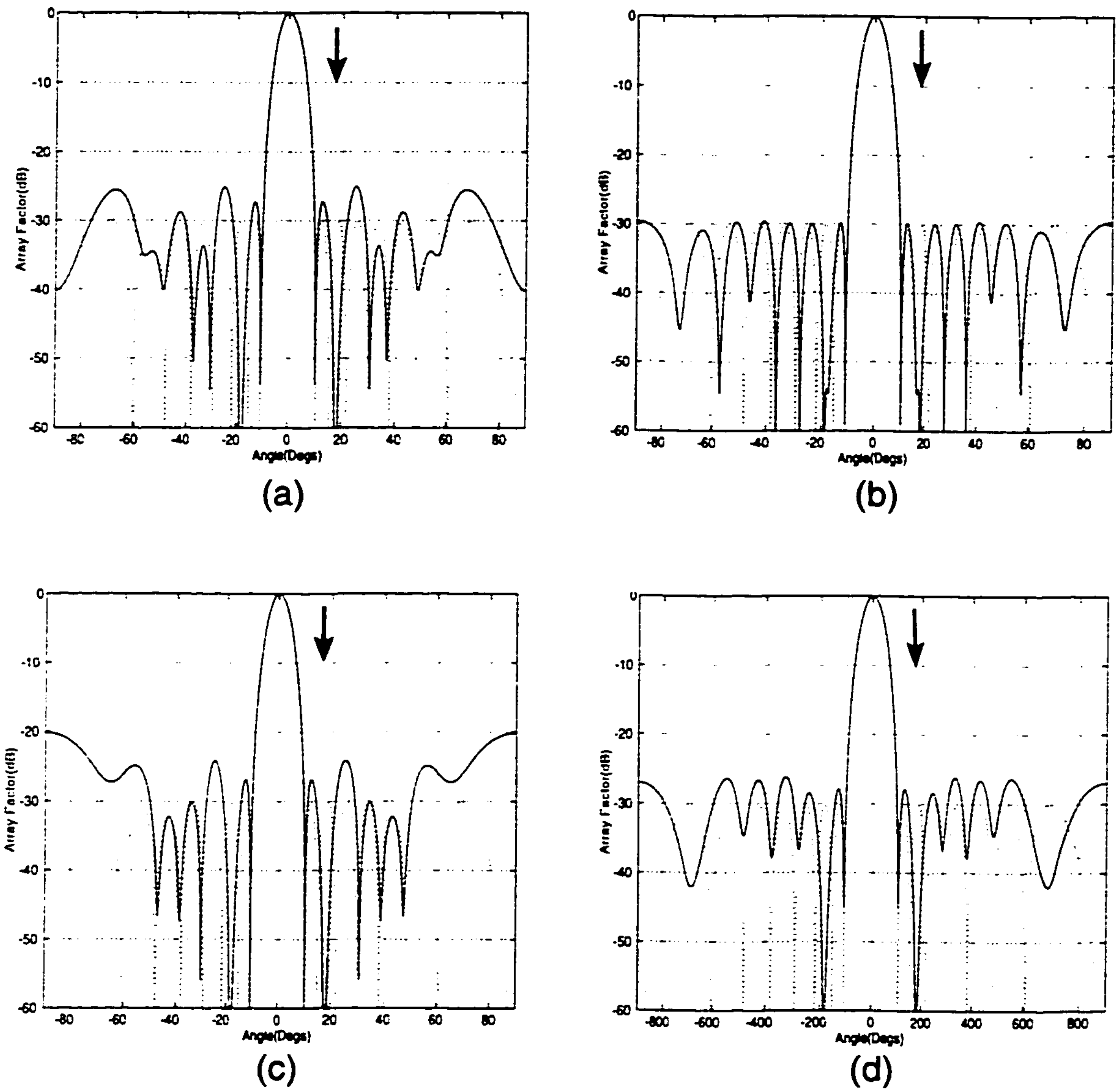


Figure 5.15: Array patterns of 16 element chebyshev array with -30 dB side lobe level and one null imposed on the peak of second sidelobe at 18.4° (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 0.38 dB (c)Minimum (optimum) number of elements are controlled (K=4) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=7) with sidelobes restricted to 3.48 dB

ELEMENT Number	Fig. 5.15(a) Full(WOSR) Δ_n	Fig. 5.15(b) Full(WSR) Δ_n	Fig. 5.15(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.15(d). Partial(WSR) $\hat{\Delta}_n$
1	-0.0117	-0.0853	0	0
2	-0.0004	0.0001	0	0
3	0.0002	0.0166	0	0.0624
4	0.0277	0.0257	0.0289	0.0701
5	0.0009	0.0000	0	0
6	-0.0148	-0.0136	0	0
7	-0.0358	-0.0177	-0.0651	-0.0524
8	-0.0349	-0.0106	0	0
9	0.0294	0.0007	0	0
10	0.0211	0.0145	0.0619	0
11	0.0176	0.0050	0	0
12	-0.0292	-0.0106	0	0
13	-0.0347	-0.0263	-0.0471	-0.0433
14	-0.0274	-0.0467	0	-0.0565
15	-0.0265	0.0067	0	0
16	0.0252	0.1306	0	0.0409

Table 5.29: Computed element position perturbations for Fig. 5.15 when a single null is steered at 18.4°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.15(a) Full(WOSR)	Fig. 5.15(b) Full(WSR)	Fig. 5.15(c) Partial(WOSR)	Fig. 5.15(d) Partial(WSR)
Min. Controlled Elements K		16	16	4	6
SLV (dB)		4.9133	0.3852	9.8298	3.8466
DIRECTIVITY	13.7874	13.7283	13.8069	13.6434	13.6790
HPBW (DEG.)	7.9793	7.9896	7.9623	7.9776	8.0308
SLL (dB)	-30	-25.0867	-29.6148	-20.1702	-26.1534
Null Depth(dB)	-30	-69.54	-60.00	-62.76	-66.66
No. of Generations		44	153	82	97
CPU time (Sec)		14.6	612	27.3	388

Table 5.30: Computed Array Parameters for Fig. 5.15 when a single null is steered at 18.4°.

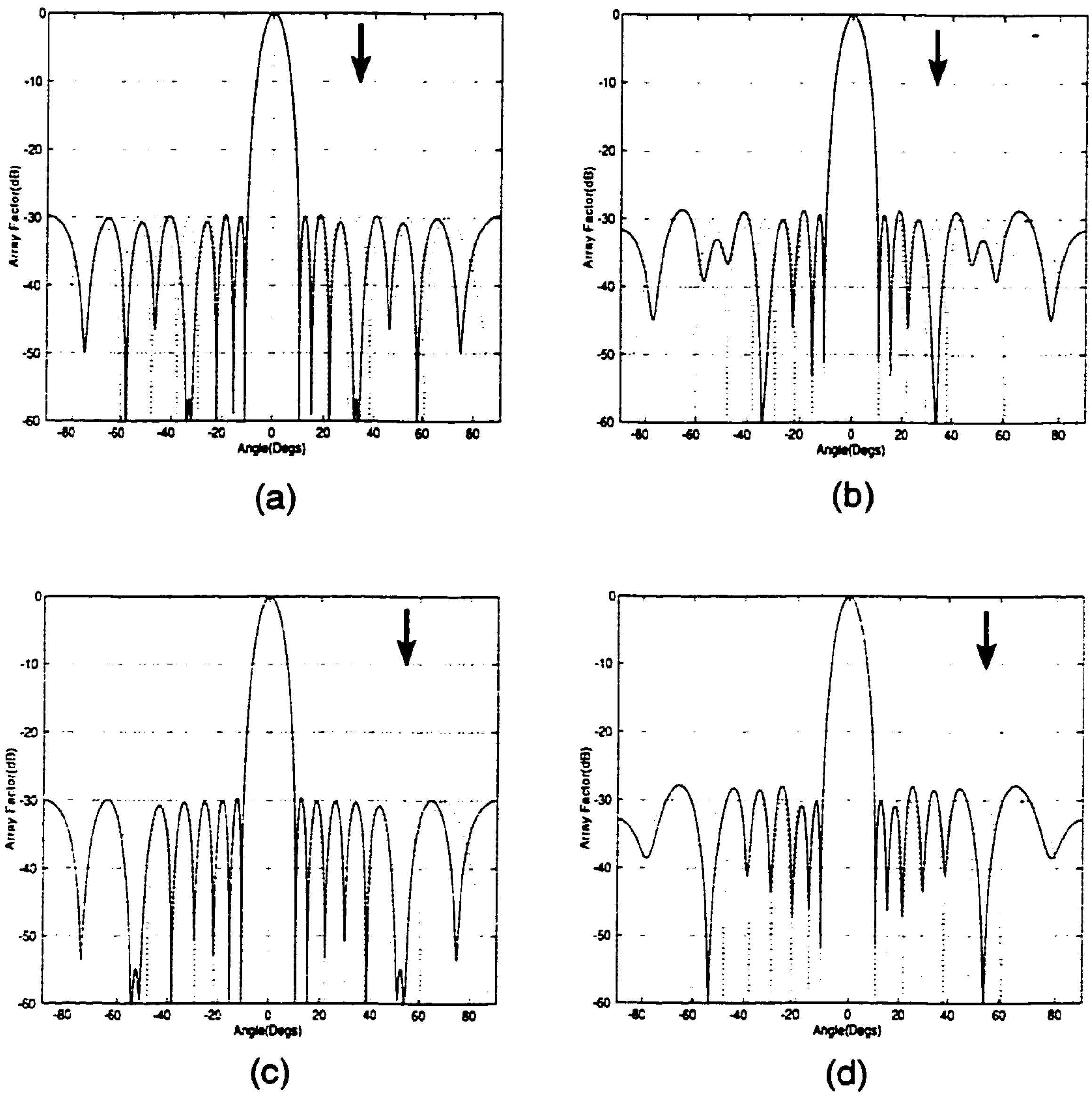


Figure 5.16: Array patterns of 16 element chebyshev array with -30 dB sidelobe level with (a) one null imposed on the peak of fourth sidelobe at 33.7° when all elements are controlled and sidelobes restricted to 0.40 dB (b) Minimum (optimum) number of elements are controlled (K=5) with sidelobes restricted to 1.34 dB (c) one null imposed on the peak of sixth sidelobe at 53.9° when all elements are controlled and sidelobes restricted to 0.38 dB (d) Minimum (optimum) number of elements are controlled (K=4) with sidelobes restricted to 2.16 dB

perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.16(a) when all the 16-elements are perturbed and the sidelobe variation is restricted to 0.40 dB. Fig.5.16(b) shows the pattern when the sidelobe variation is restricted to 1.34 dB while achieving the required null. K is equal to 5 in this case. Fig.5.16(c) shows the resulting pattern when all the 16-elements are perturbed and the sidelobe variation is restricted to 0.38 dB. Fig.5.16(d) shows the pattern when the sidelobe variation is restricted to 2.16 dB while achieving the required null. K is equal to 4 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.16 are given in table 5.31 and table 5.32 respectively.

In general for the case of a single null in 16-element chebyshev partially adaptive arrays with sidelobe restriction, the same behavior obtained in case of 8-element arrays is noticed here. It is observed that as we steer a single null towards the main beam, the minimum no of controlled elements K and the SLV is increase, while the other array parameters such as directivity and HPBW remain almost unchanged. This is due to the reason that steering of a null near the main beam requires a higher order of degree.s of freedom.

It is also observed that, when the sidelobe restriction is applied, the number of controlled elements is increasing from the minimum possible value, which is due to increase in the number of constraints in this optimization. Tables 5.30, and 5.32, show a comparison of array parameters such as directivity and HPBW. The results show that they are almost unchanged. The sidelobe variation (SLV) and sidelobe

ELEMENT Number	Fig. 5.16(a) (Null at 33.7°) Full(WSR) Δ_n	Fig. 5.16(b) (Null at 33.7°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.16(c) (Null at 53.9°) Full(WSR) Δ_n	Fig. 5.16(d) (Null at 53.9°) Partial(WSR) $\hat{\Delta}_n$
1	-0.0370	0	-0.0007	0
2	0.0540	0.0588	0.0266	0
3	-0.0021	0	-0.0235	0
4	-0.0173	0	0.0141	0
5	0.0010	0	-0.0040	0
6	0.0160	0	0.0004	0
7	-0.0085	0	0.0039	0
8	-0.0055	-0.0170	0.0060	0
9	0.0049	0	-0.0011	0.0142
10	0.0009	0	-0.0069	0
11	-0.0131	-0.0342	0.0005	0
12	-0.0003	0	0.0120	0
13	0.0098	0.0171	-0.0122	-0.0454
14	0.0001	0	0.0156	0.0449
15	-0.0502	-0.0746	-0.0314	-0.0149
16	0.0137	0	0.0131	0

Table 5.31: Computed element position perturbations for Fig. 5.16 as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.16(a) (Null at 33.7°) Full(WSR)	Fig. 5.16(b) (Null at 33.7°) Partial(WSR)	Fig. 5.16(c) (Null at 53.9°) Full(WSR)	Fig. 5.16(d) (Null at 53.9°) Partial(WSR)
Min. Controlled Elements K		16	5	16	4
SLV (dB)		0.4088	1.3498	0.3887	2.1682
DIRECTIVITY	13.7874	13.7831	13.7562	13.7967	13.8009
HPBW (DEG.)	7.9793	7.9877	8.0097	7.9811	7.9835
SLL (dB)	-30	-29.5912	-28.6502	-29.6113	-27.8318
Null Depth(dB)	-30	-60.13	-60.43	-61.26	-60.40
No. of Generations		87	89	61	83
CPU time (Sec)		348	356	244	332

Table 5.32: Computed Array Parameters for Fig. 5.16

level (SLL) in case of partially adaptive arrays are slightly higher than in fully adaptive case, since it is obvious that, the best array parameters are obtained when all the 16-elements are perturbed, as it affords the greatest control over the array response.

The results of Fig.5.17 (a) and (b) show two nulls, which has been steered to the peaks of the second and fourth sidelobe levels at 18.4° and 33.7° respectively. The results of Fig.5.17 (c) and (d) show three nulls, which has been steered to the peaks of the second, fifth and seventh sidelobe levels at 18.4°, 42.9° and 69.4° respectively. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.17(a) when all the 16-elements are perturbed and the sidelobe variation is restricted to 1.46 dB. Fig.5.17(b) shows the pattern when the sidelobe variation is restricted to 2.24 dB while achieving the required nulls. K is equal to 8 in this case. Fig.5.17(c) shows the resulting pattern when all the 16-elements are perturbed and the sidelobe variation is restricted to 3.81 dB. Fig.5.17(d) shows the pattern when the sidelobe variation is restricted to 4.47 dB while achieving the required nulls. K is equal to 10 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.17 are given in table 5.33 and table 5.34 respectively.

The results of Fig.5.18 show four nulls, which has been steered to the peaks of the second, fourth, sixth and seventh sidelobe levels at 18.4°, 33.7°, 53.9° and 69.4° respectively. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.18(a) when all the 16-elements are perturbed without restricting

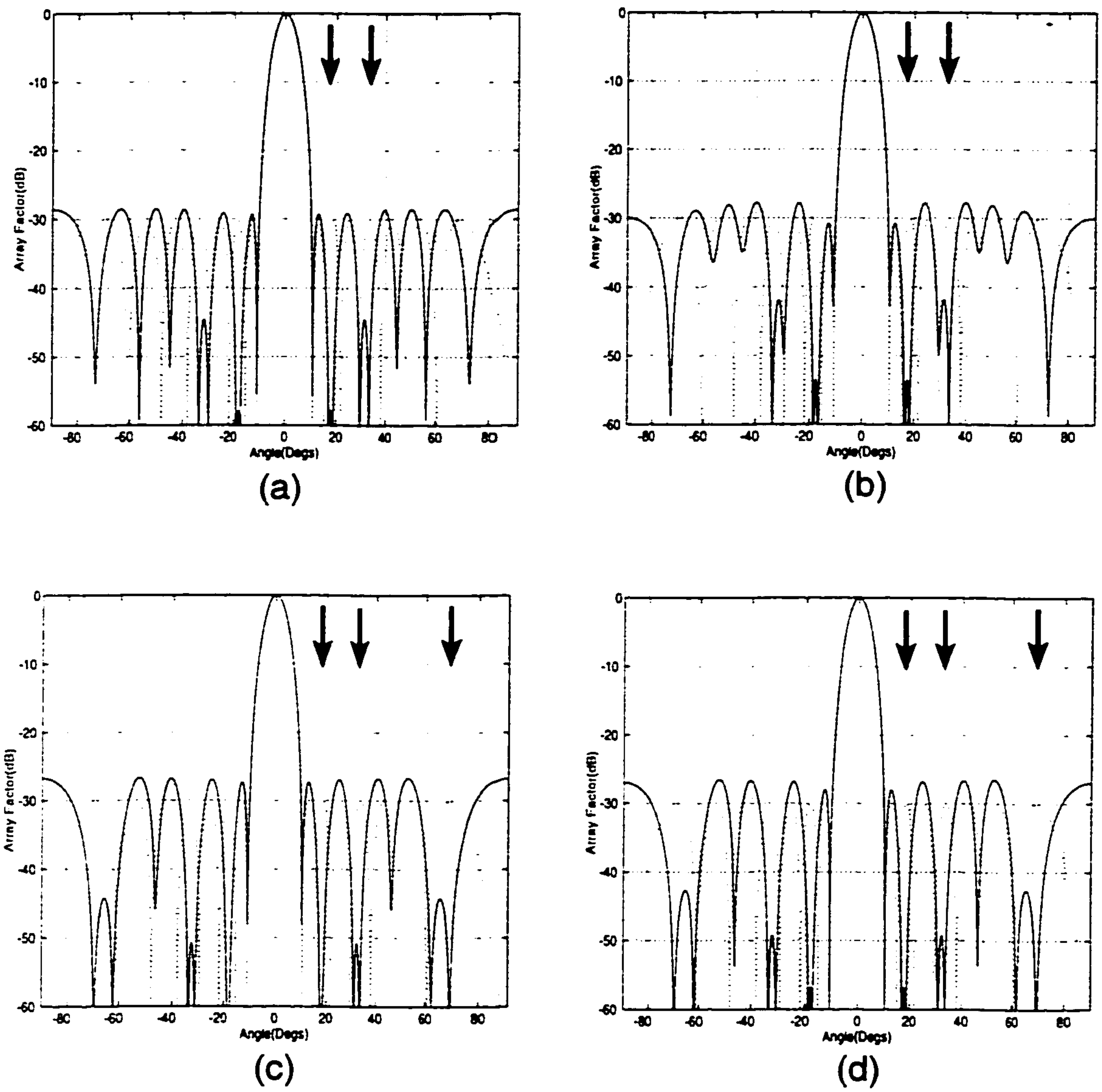


Figure 5.17: Array patterns of 16 element chebyshev array with -30 dB sidelobe level with (a) two nulls imposed at 18.4° and 33.7° respectively, when all elements are controlled and sidelobes restricted to 1.46 dB (b) Minimum (optimum) number of elements are controlled (K=8) with sidelobes restricted to 2.24 dB (c) three nulls imposed at 18.4°, 42.9° and 69.4° respectively, when all elements are controlled and sidelobes restricted to 3.81 dB (d) Minimum (optimum) number of elements are controlled (K=11) with sidelobes restricted to 4.47 dB

ELEMENT Number	Fig. 5.17(a) (Nulls at 18.4°, 33.7°) Full(WSR) Δ_n	Fig. 5.17(b) (Nulls at 18.4°, 33.7°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.17(c) (Nulls at 18.4°, 33.7°,69.4°) Full(WSR) Δ_n	Fig. 5.17(d) (Nulls at 18.4°, 33.7°,69.4°) Partial(WSR) $\hat{\Delta}_n$
1	-0.1131	-0.1135	-0.0409	-0.0749
2	0.0257	0	0.0286	0
3	0.0260	0.0211	0.0295	0.0182
4	0.0104	0	0.0083	0
5	0.0133	0	-0.0003	0
6	-0.0007	0	-0.0006	0
7	-0.0248	-0.0473	-0.0470	-0.0431
8	-0.0132	-0.0396	-0.0281	-0.0158
9	0.0187	0	0.0083	0.0207
10	0.0234	0	0.0457	0.0482
11	0.0003	0	-0.0009	0
12	-0.0162	-0.0228	-0.0009	0
13	-0.0175	-0.0168	-0.0024	0
14	-0.0296	0	-0.0274	-0.0232
15	-0.0399	0.0196	-0.0049	0
16	0.1038	0.1587	0.0805	0.0816

Table 5.33: Computed element position perturbations for Fig. 5.17 as a function of λ.

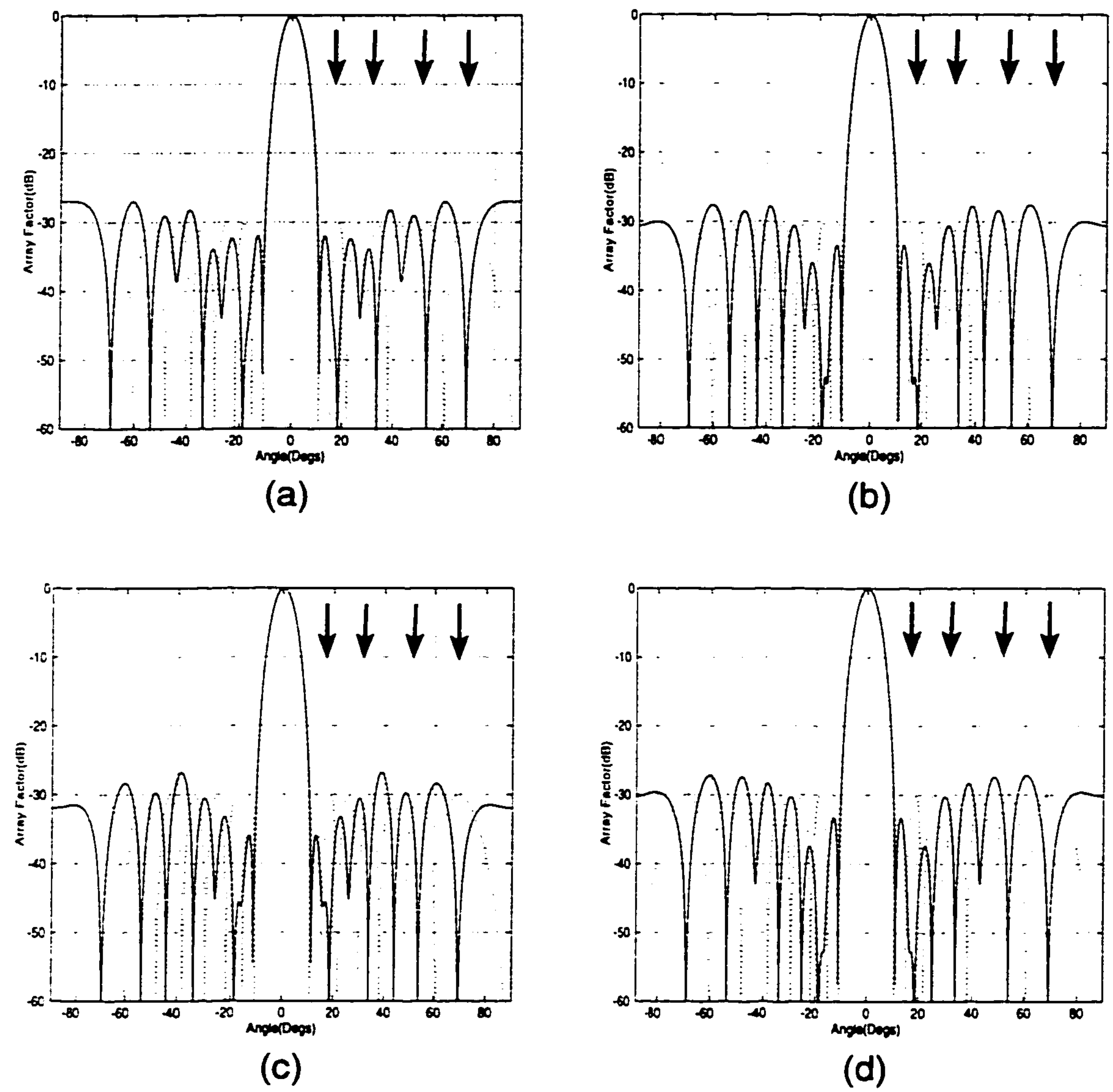


Figure 5.18: Array patterns of 16 element chebyshev array of -30 dB sidelobe level with four nulls imposed at 18.4°, 33.7°, 53.9° and 69.4° (peak of 2^{nd}, 4^{th}, 6^{th} and 7^{th} sidelobes). (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 2.45 dB (c)Minimum (optimum) number of elements are controlled (K=7) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=9) with sidelobes restricted to 2.81 dB

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.17(a) (Nulls at 18.4°, 33.7°) Full(WSR)	Fig. 5.17(b) (Nulls at 18.4°, 33.7°) Partial(WSR)	Fig. 5.17(c) (Nulls at 18.4°, 33.7°,69.4°) Full(WSR)	Fig. 5.17(d) (Nulls at 18.4°, 33.7°,69.4°) Partial(WSR)
Min. Controlled Elements K		16	8	16	8
SLV (dB)		1.4641	2.2459	3.3042	3.4861
DIRECTIVITY	13.7874	13.7729	13.8537	13.7822	13.8105
HPBW (DEG.)	7.9793	7.9736	7.9301	7.9610	7.9400
SLL (dB)	-30	-28.5359	-27.7541	-26.6958	-26.5139
Null Depth(dB)	-30	-60.73	-65.08	-60.20	-60.16
No. of Generations		171	200	126	116
CPU time (Sec)		684	800	504	464

Table 5.34: Computed Array Parameters for Fig. 5.17

the sidelobe variation. Fig.5.18(b) shows the perturbed pattern compared to the initial, when all the 16-elements are perturbed and the sidelobe variation is restricted to 2.45 dB. Fig.5.18(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=7 in this case. The required null has been achieved precisely, but the sidelobe level variation has changed to more than 3 dB. Fig.5.18(d) shows the pattern when the sidelobe variation is restricted to 2.81 dB while achieving the required null. K is equal to 9 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.18 are given in table 5.35 and table 5.36 respectively.

For the case of multiple nulls in 16-element chebyshev partially arrays with sidelobe restriction, the same behavior obtained in case of 8-element array is noticed here. It is observed that, as we increased the number of nulls from one to four,

ELEMENT Number	Fig. 5.18(a) Full(WOSR) Δ_n	Fig. 5.18(b) Full(WSR) Δ_n	Fig. 5.18(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.18(d) Partial(WSR) $\hat{\Delta}_n$
1	-0.1329	-0.2543	-0.3030	-0.2573
2	0.0160	-0.0170	-0.0622	0
3	0.0079	0.0314	0.0165	0.0323
4	0.0033	0.0054	0	0
5	-0.0008	-0.0002	0	0.0053
6	-0.0023	-0.0053	0	-0.0016
7	-0.0005	-0.0010	0	0
8	-0.0116	-0.0010	0	0
9	0.0026	0.0079	0.0134	0.0039
10	0.0145	0.0008	0	0
11	0.0037	0.0031	0	0.0102
12	-0.0100	0.0006	0	0
13	-0.0142	0.0001	0	-0.0072
14	-0.0348	-0.0297	-0.0262	-0.0303
15	0.0005	-0.0001	0.0282	0
16	0.2572	0.1901	0.1681	0.1951

Table 5.35: Computed element position perturbations for Fig. 5.18 when four nulls are steered at 18.4°,33.7°,53.9° and 69.4°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.18(a) Full(WOSR)	Fig. 5.18(b) Full(WSR)	Fig. 5.18(c) Partial(WOSR)	Fig. 5.18(d) Partial(WSR)
Min. Controlled Elements K		16	16	7	9
SLV (dB)		2.9996	2.4525	3.2134	2.8176
DIRECTIVITY	13.7874	13.8575	13.8610	13.9164	13.8484
HPBW (DEG.)	7.9793	7.9239	7.9074	7.8837	7.9121
SLL (dB)	-30	-17.0004	-27.5475	-26.7866	-27.1824
Null Depth(dB)	-30	-60.13	-60.04	-60.09	-60.13
No. of Generations		585	186	296	368
CPU time (Sec)		195	744	98.6	1472

Table 5.36: Computed Array Parameters for Fig. 5.18 when four nulls are steered at 18.4°,33.7°,53.9° and 69.4°.

the minimum no of controlled elements K, SLV and SLL has also increased. From the tables 5.34 and 5.36 comparing the array parameters of partially adaptive array, with the corresponding fully adaptive array, when multiple nulls are imposed, one notices that directivity and HPBW are almost unchanged. The sidelobe variation (SLV) and SLL in case of partially adaptive arrays is slightly higher than in fully adaptive case, since it is obvious that the best array parameters are obtained, when all the 16-elements are perturbed, as it affords the greatest control over the array response.

5.5.3 Determination of the realizable minimum number of controlled elements

To implement this technique for a single null, the null location which requires the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steering. In the case of 16-element chebyshev partially adaptive arrays, a null on the peak of second sidelobe requires the use of 6 controlled elements. Now keeping the location of these 6 controlled elements fixed, a single null is imposed on the peak of fifth sidelobe as shown in Fig.5.19(b) with sidelobes restricted to 1.94 dB . A single null is imposed on the peak of sixth sidelobe as shown in Fig.5.19(c) with sidelobes restricted to 1.61 dB and a single null is imposed on the peak of seventh sidelobe

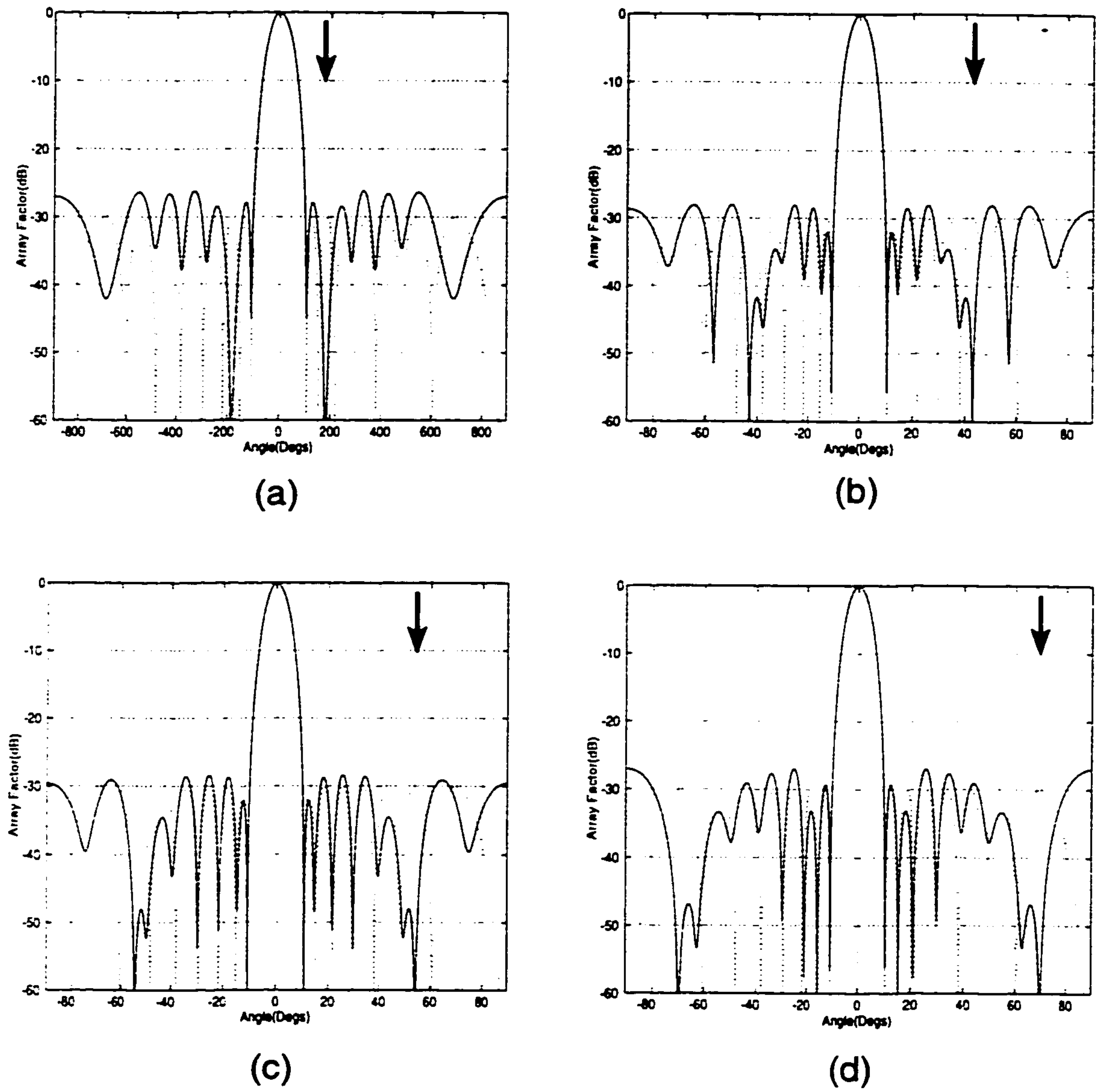


Figure 5.19: Array patterns of 16 element chebyshev array with -30 dB sidelobe level, perturbing 6 fixed elements with (a) one null imposed on the peak of second sidelobe at 18.4° (b) one null imposed on the peak of fourth sidelobe at 33.7° (c) one null imposed on the peak of sixth sidelobe at 53.9° (d) one null imposed on the peak of seventh sidelobe at 69.4°

as shown in Fig.5.19(c) with sidelobes restricted to 3 dB. Therefore perturbing only these 6 fixed elements out of 16-elements, we can steer a single null any where in the sidelobe region of an 16-element chebyshev partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.19 are given in table 5.37 and table 5.38 respectively.

The results given in table 5.38 show that the sidelobe variation (SLV) when the

ELEMENT Number	Fig. 5.19(a) (Null at 18.4°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.19(b) (Null at 42.9°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.19(c) (Null at 53.9°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.19(d) (Null at 69.4°) Partial(WSR) $\hat{\Delta}_n$
3	0.0624	-0.0633	-0.0456	-0.0126
4	0.0701	0.0127	-0.0007	0.0214
7	-0.0524	-0.0157	0.0129	-0.0319
13	-0.0433	0.0220	-0.0104	-0.0362
14	-0.0565	0.0554	0.0575	0.0168
16	0.0409	0.1474	0.0536	-0.0138

Table 5.37: Computed element position perturbations for Fig. 5.19 as a function of λ.

null is imposed at fifth, sixth or seventh sidelobe, is higher than the previous results inspite of increasing the no. of controlled elements. This shows that, for a particular null location the SLV depends on the location of the minimum controlled elements.

To implement this technique for two nulls, the null locations which requires the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steering. In the case of 16-element chebyshev partially adaptive arrays two nulls

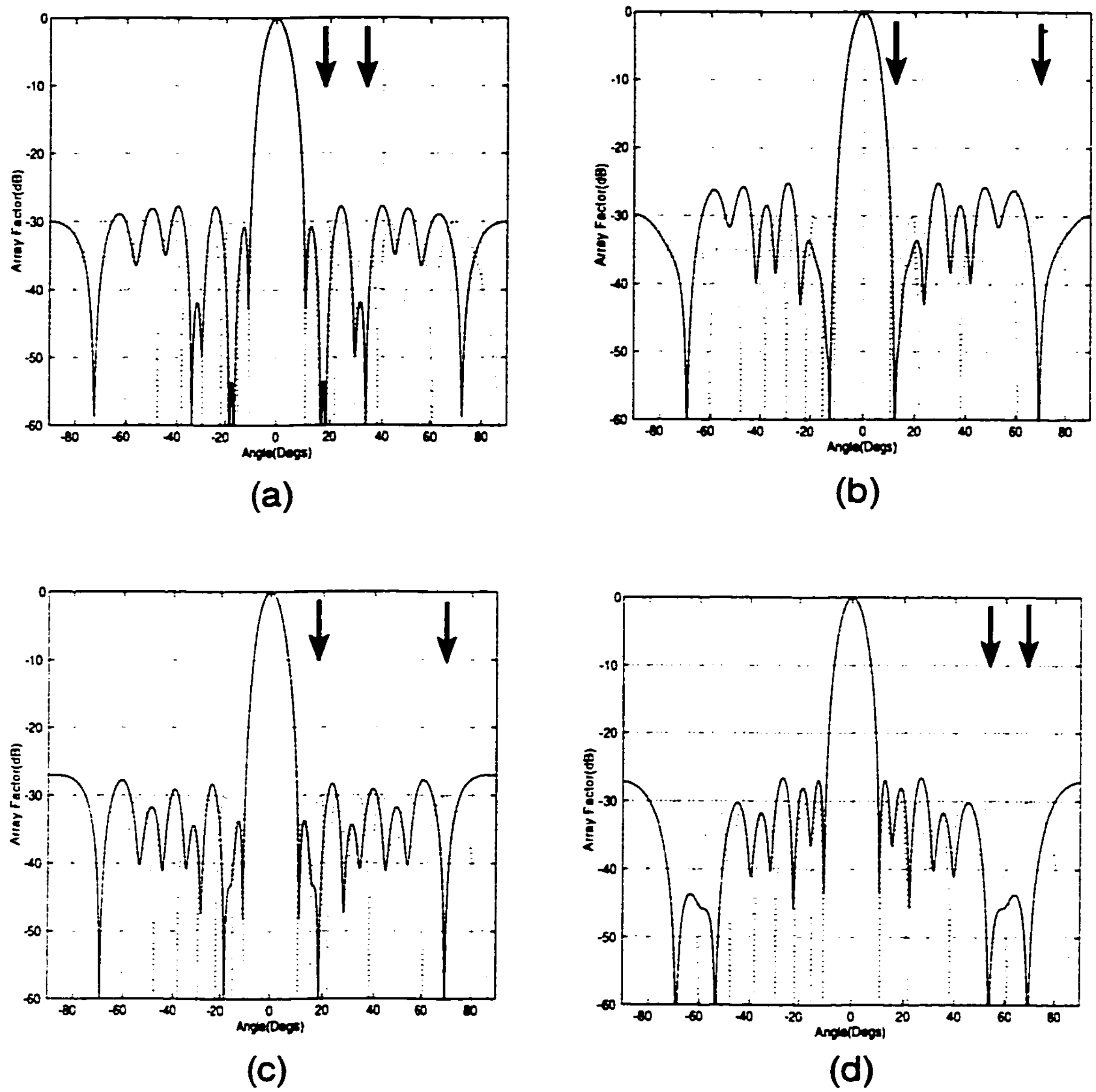


Figure 5.20: Array patterns of 16 element chebyshev array with -30 dB sidelobe level, perturbing 8 fixed elements with (a) two nulls imposed on the peak of second and fourth sidelobes at 18.4° and 33.7° (b) two nulls imposed on the peak of first and sixth sidelobes at 12.6° and 69.4° (c) two nulls imposed on the peak of second and seventh sidelobes at 18.4° and 69.4° (d) two nulls imposed on the peak of sixth and seventh sidelobes at 53.9° and 69.4°

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.19(a) (Null at 18.4°) Partial (WSR)	Fig. 5.19(b) (Null at 42.9°) Partial (WSR)	Fig. 5.19(c) (Null at 53.9°) Partial (WSR)	Fig. 5.19(d) (Null at 69.4°) Partial (WSR)
No. of Controlled Elements		6	6	6	6
SLV (dB)		3.8466	1.9463	1.6174	3.0034
DIRECTIVITY	13.7874	13.6790	13.9674	13.8451	13.7829
HPBW (DEG.)	7.9793	8.0308	7.8896	7.9463	7.9860
SLL (dB)	-30	-26.5134	-28.0537	-28.3826	-26.9966
Null Depth(dB)	-30	-66.66	-60.87	-60.35	-61.74
No. of Generations		97	359	462	58
CPU time (Sec)		388	2332	640	2436

Table 5.38: Computed Array Parameters for Fig. 5.19

on the peak of second and fourth sidelobe requires the use of 8 controlled elements. Now keeping the location of these 8 controlled elements fixed, two nulls are imposed on the peak of first and seventh sidelobes as shown in Fig.5.20(b) with sidelobes restricted to 4.83 dB . Two nulls are imposed on the peak of second and seventh sidelobes as shown in Fig.5.20(c) with sidelobes restricted to 2.95 dB and two nulls are imposed on the peak of sixth and seventh sidelobe as shown in Fig.5.20(c) with sidelobes restricted to 3.41 dB. Therefore perturbing only these 8 fixed elements out of 16-elements, we can steer two nulls any where in the sidelobe region of an 16-element chebyshev partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.20 are given in table 5.39 and table 5.40 respectively.

The 16-element chebyshev partially adaptive array is suitable for steering upto four

ELEMENT Number	Fig. 5.20(a) (Nulls at 18.4°,33.7°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.20(b) (Nulls at 12.6°,69.4°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.20(c) (Nulls at 18.4°,69.4°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.20(d) (Nulls at 53.9°,69.4°) Partial(WSR) $\hat{\Delta}_n$
1	-0.1135	-0.3628	-0.1111	0.0000
3	0.0211	0.0357	-0.0089	-0.0568
7	-0.0473	0.0430	-0.0264	-0.0302
8	-0.0396	-0.0044	-0.0289	-0.0130
12	-0.0228	-0.0192	-0.0073	0.0189
13	-0.0168	-0.0134	-0.0275	-0.0236
15	0.0196	0.1595	0.0704	-0.1394
16	0.1587	0.3086	0.2637	-0.1402

Table 5.39: Computed element position perturbations for Fig. 5.20 as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.20(a) (Nulls at 18.4°, 33.7°) Partial (WSR)	Fig. 5.20(b) (Nulls at 12.6°, 69.4°) Partial (WSR)	Fig. 5.20(c) (Nulls at 18.4°, 69.4°) Partial (WSR)	Fig. 5.20(d) (Nulls at 53.9°, 69.4°) Partial (WSR)
No. of Controlled Elements		8	8	8	8
SLV (dB)		2.2459	4.8357	2.9503	3.4154
DIRECTIVITY	13.7874	13.8537	13.9340	13.9089	13.7725
HPBW (DEG.)	7.9793	7.9301	7.8501	7.8962	8.0127
SLL (dB)	-30	-27.7541	-25.1643	-27.0497	-26.5846
Null Depth(dB)	-30	-65.08	-61.70	-61.84	-62.42
No. of Generations		200	623	461	212
CPU time (Sec)		800	2492	1844	848

Table 5.40: Computed Array Parameters for Fig. 5.20

nulls, because steering five nulls requires more than 9 controlled elements out of 16.

5.6 Simulation results of 30 element uniform partially adaptive arrays

5.6.1 Introduction

In this section, we study the array performance and behavior of its parameters such as half-power beam width (HPBW), directivity , sidelobe level (SLL), the minimum number of controlled elements K and the sidelobe variation (SLV) on 30-element uniform array with element spacing of 0.5 λ.

The validity of the proposed partially adaptive method is examined by first placing a single null and then placing up to six multiple nulls on the peaks of sidelobes. The results are compared with the fully adaptive case with and without sidelobe restrictions.

5.6.2 Simulation results

The results of Fig.5.21 show one null, which has been steered to the peak of the second sidelobe level at 9.4°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.21(a) when all the 30-elements are perturbed without restricting the sidelobe variation. Fig.5.21(b) shows the perturbed pattern compared

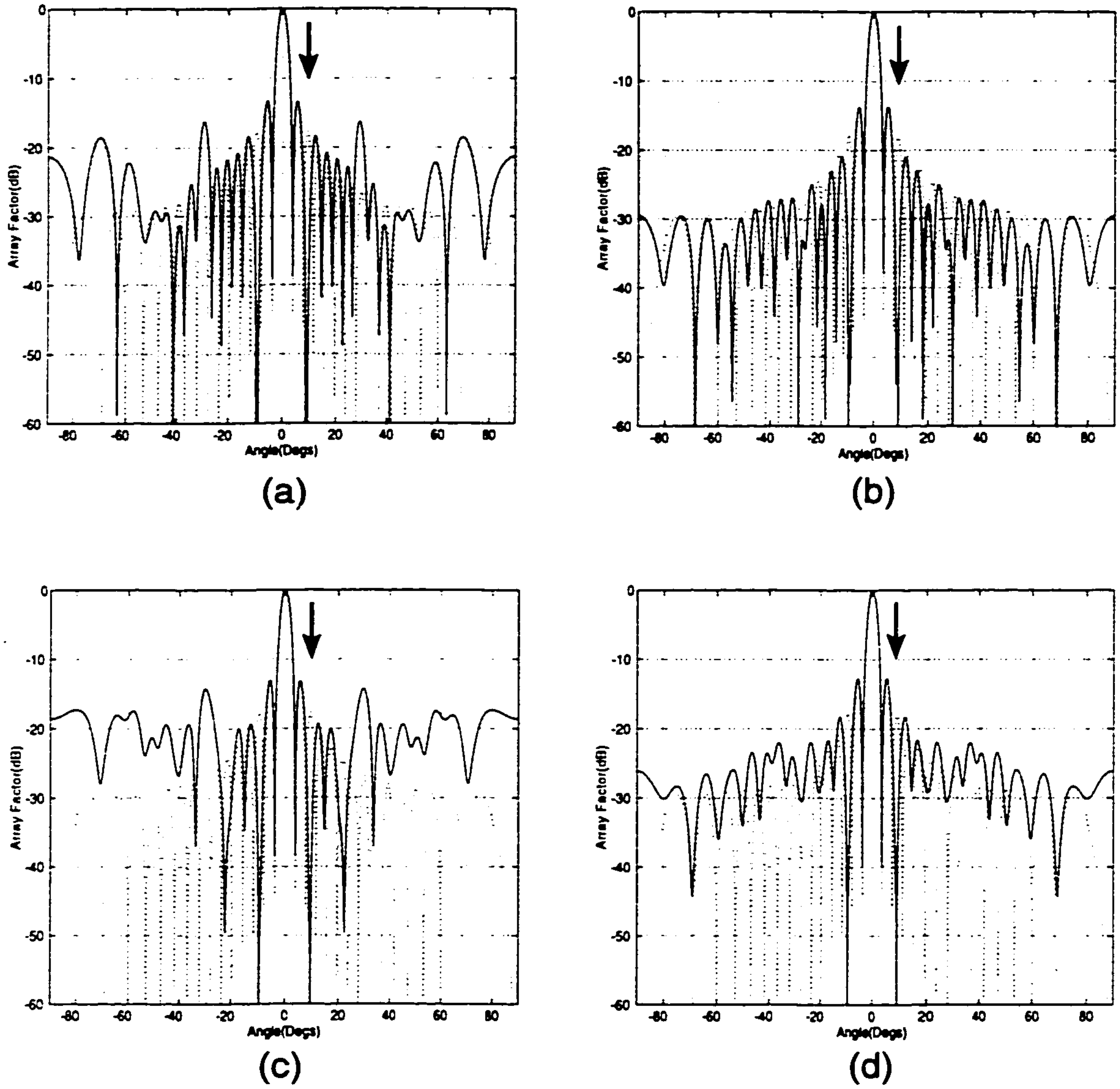


Figure 5.21: Array patterns of 30 element uniform array with one null imposed on the peak of second sidelobe at 9.4° (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 0.62 dB (c)Minimum (optimum) number of elements are controlled (K=10) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=16) with sidelobes restricted to 6.02 dB

to the initial, when all the 30-elements are perturbed and the sidelobe variation is restricted to 0.62 dB. Fig.5.21(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=10 in this case. The required null has been achieved precisely, but the sidelobe level variation has changed to more than 12 dB. Fig.5.21(d) shows the pattern when the sidelobe variation is restricted to 6.02 dB while achieving the required null. K is equal to 16 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.21 are given in table 5.41 and table 5.42 respectively.

The results of Fig.5.22 (a) and (b) show one null, which has been steered to the peak of the sixth sidelobe level at 25.6° and the results of Fig.5.22 (c) and (d) show one null, which has been steered to the peak of the eleventh sidelobe level at 50°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.22(a) when all the 30-elements are perturbed and the sidelobe variation is restricted to 0.40 dB. Fig.5.22(b) shows the pattern when the sidelobe variation is restricted to 1.41 dB while achieving the required null. K is equal to 10 in this case. Fig.5.22(c) shows the resulting pattern when all the 30-elements are perturbed and the sidelobe variation is restricted to 0.50 dB. Fig.5.22(d) shows the pattern when the sidelobe variation is restricted to 1.00 dB while achieving the required null. K is equal to 6 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.22 are given in table 5.43 and table 5.44 respectively.

In general for the case of a single null in 30-element uniform partially adaptive

ELEMENT Number	Fig. 5.21(a) Full(WOSR) Δ_n	Fig. 5.21(b) Full(WSR) Δ_n	Fig. 5.21(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.21(d) Partial(WSR) $\hat{\Delta}_n$
1	-0.2042	-0.3828	-0.3570	-0.3541
2	-0.2065	-0.1677	0	-0.1115
3	-0.1608	-0.0000	0	0
4	0.1915	0.0784	0	0
5	0.2101	0.2158	0.3860	0
6	0.2211	0.2555	0.4095	0
7	0.1803	0.1845	0.3787	0
8	0.1686	0.1118	0	0
9	0.0615	0.0278	0	0
10	-0.1236	-0.0392	0	0
11	-0.1626	-0.1312	0	-0.1545
12	-0.2298	-0.2054	-0.4431	-0.3260
13	-0.1704	-0.2526	-0.4025	-0.3222
14	-0.2194	-0.1397	0	0
15	-0.1932	-0.0419	0	0
16	0.1036	0.0055	0	0
17	0.2264	0.1203	0	0.2638
18	0.2058	0.1717	0.4077	0.4179
19	0.1847	0.2070	0.4116	0.4288
20	0.1995	0.1460	0	0.2902
21	0.1466	0.0595	0	0.1904
22	-0.0855	-0.0530	0	0
23	-0.2328	-0.1138	0	-0.2193
24	-0.1998	-0.1831	0	-0.2093
25	-0.2087	-0.1984	-0.4119	-0.3200
26	-0.1775	-0.2256	0	-0.2935
27	-0.2069	-0.1311	0	0
28	0.1399	0.0418	0	0
29	0.2482	0.2732	0	0.2098
30	0.1858	0.4672	0.3895	0.4008

Table 5.41: Computed element position perturbations for Fig. 5.21 when a single null is steered at 9.4°. Perturbations are given as a function of λ.

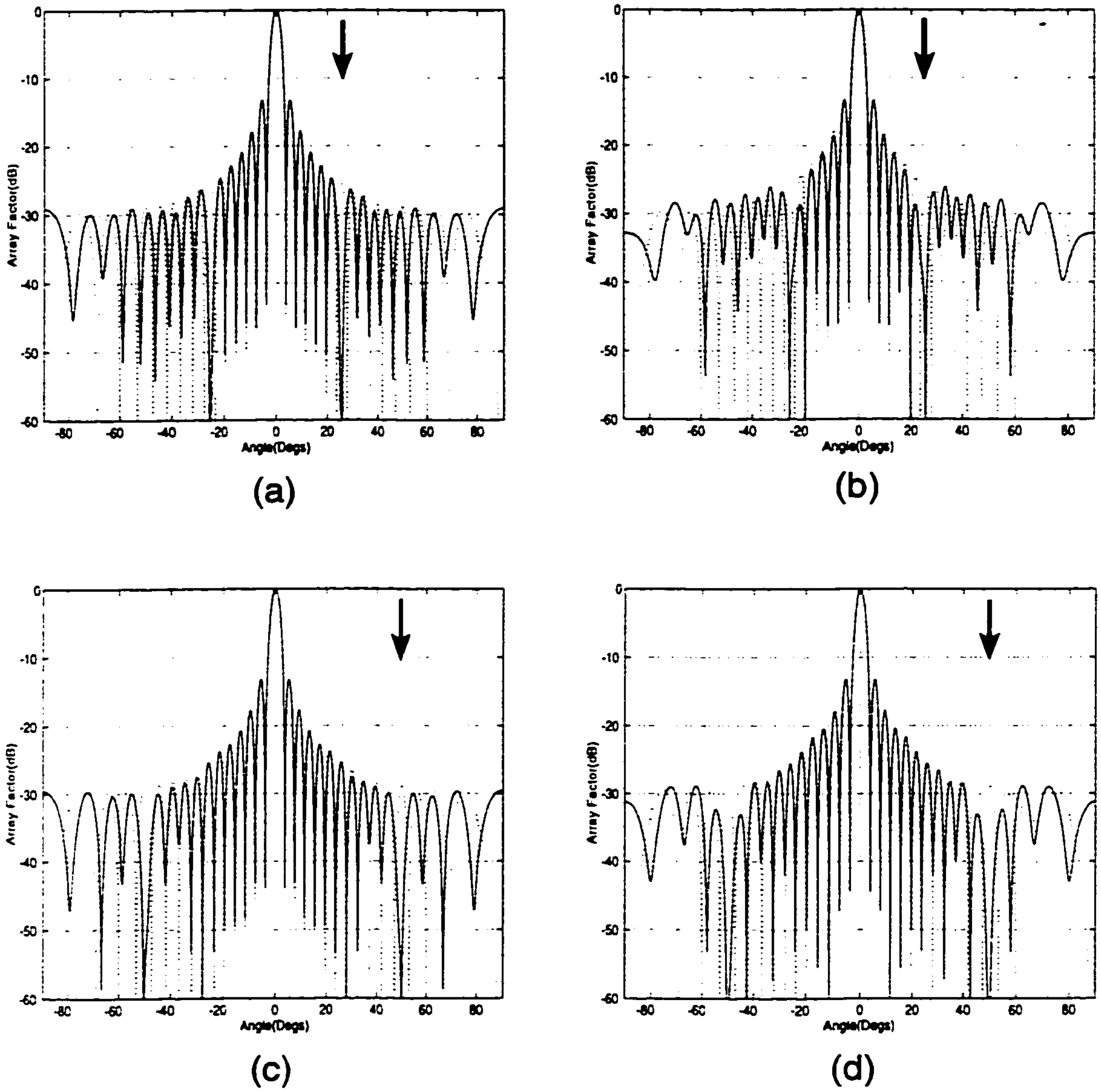


Figure 5.22: Array patterns of 30 element uniform array with (a) one null imposed on the peak of sixth sidelobe at 25.6° when all elements are controlled and sidelobes restricted to 0.40 dB (b) Minimum (optimum) number of elements are controlled (K=10) with sidelobes restricted to 1.41 dB (c) one null imposed on the peak of eleventh sidelobe at 50° when all elements are controlled and sidelobes restricted to 0.50 dB (d) Minimum (optimum) number of elements are controlled (K=6) with sidelobes restricted to 1.00 dB

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.21(a) Full(WOSR)	Fig. 5.21(b) Full(WSR)	Fig. 5.21(c) Partial(WOSR)	Fig. 5.21(d) Partial(WSR)
Min. Controlled Elements K		30	30	10	16
SLV (dB)		10.98	1.2228	12.24	6.0276
DIRECTIVITY	30	28.5811	30.8407	27.8331	30.6171
HPBW (DEG.)	3.3860	3.3814	3.3697	3.3832	3.3441
SLL (dB)	-13.23	-13.3365	-13.7487	-13.1288	-12.7403
Null Depth(dB)	-17.73	-66.12	-60.03	-61.23	-67.19
No. of Generations		265	1947	486	501
CPU time (Sec)		88.3	7788	162	2004

Table 5.42: Computed Array Parameters for Fig. 5.21 when a single null is steered at 9.4°.

arrays with sidelobe restriction, the same behavior obtained in case of 8 and 16 element arrays is noticed here. It is observed that as we steer a single null towards the main beam, the minimum no of controlled elements K and the SLV increase. The reason for this behavior is that as we are steering a null from a lower energy concentrated area to a higher energy concentrated area, it requires a higher order of degrees of freedom.

It is also observed that, when the sidelobe restriction is applied, the number of controlled elements increase from the minimum possible value. This is due to increasing the number of constraints in this optimization problem. Tables 5.42, and 5.44, show a comparison of array parameters such as directivity, HPBW and SLL. The results show that they are almost unchanged. The sidelobe variation (SLV) in case of partially adaptive arrays is slightly higher than in fully adaptive case, since it is obvious that, the best array parameters are obtained, when all the 30-elements

ELEMENT Number	Fig. 5.22(a) (Null at 25.6°) Full(WSR) Δ_n	Fig. 5.22(b) (Null at 25.6°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.22(c) (Null at 50°) Full(WSR) Δ_n	Fig. 5.22(d) (Null at 50°) Partial(WSR) $\hat{\Delta}_n$
1	-0.0812	-0.1298	-0.0170	0
2	0.0228	0	0.0151	0.0393
3	0.0362	0.0129	-0.0341	-0.0361
4	0.0016	0	0.0081	0
5	-0.0187	0	0.0010	0
6	0.0009	0	0.0006	0
7	0.0495	0.0046	0.0355	0
8	0.0298	0	-0.0143	0
9	-0.0158	0	-0.0044	0
10	-0.0236	-0.0394	0.0262	0
11	0.0182	0	-0.0057	-0.0353
12	0.0366	0	0.0132	0
13	0.0155	0	0.0007	0
14	-0.0146	0	0.0003	0
15	-0.0150	0	0.0006	0
16	0.0251	0	-0.0166	0
17	0.0336	0.0350	0.0007	0
18	-0.0003	0	0.0006	0
19	-0.0445	0	-0.0193	0
20	0.0057	0	0.0006	0
21	0.0410	0	-0.0051	0
22	0.0241	0	0.0006	0
23	-0.0306	-0.0700	0.0109	0
24	-0.0216	-0.0959	0.0157	-0.0531
25	0.0104	0	-0.0002	0
26	0.0410	0.0706	0.0001	0
27	-0.0007	0	-0.0008	0
28	-0.0267	-0.0878	0.0224	0.0387
29	-0.0257	0	-0.0333	-0.0356
30	0.0401	0.1230	0.0172	0

Table 5.43: Computed element position perturbations for Fig. 5.22 as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.22(a) (Null at 25.6°) Full (WSR)	Fig. 5.22(b) (Null at 25.6°) Partial (WSR)	Fig. 5.22(c) (Null at 50°) Full (WSR)	Fig. 5.22(d) (Null at 50°) Partial (WSR)
Min. Controlled Elements K		30	10	30	6
SLV (dB)		0.4065	1.4176	0.5026	1.0044
DIRECTIVITY	30	30.1998	30.5263	30.1270	30.0406
HPBW (DEG.)	3.3860	3.3856	3.3812	3.3856	3.3860
SLL (dB)	-13.23	-13.3029	-13.4281	-13.2861	-13.2152
Initial Null Depth(dB)		-25.51	-25.51	-28.94	-28.94
Final Null Depth(dB)		-65.78	-60.00	-61.24	-69.31
No. of Generations		105	189	41	58
CPU time (Sec)		105	756	164	232

Table 5.44: Computed Array Parameters for Fig. 5.22

are perturbed, as it affords the greatest control over the array response.

The results of Fig.5.23 (a) and (b) show two nulls, which has been steered to the peaks of the second and fourth sidelobe levels at 9.4° and 17.4° respectively. The results of Fig.5.23 (c) and (d) show three nulls, which has been steered to the peaks of the second, fifth and eighth sidelobe levels at 9.4°, 21.4° and 34.5° respectively. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.23(a) when all the 30-elements are perturbed and the sidelobe variation is restricted to 1.52 dB. Fig.5.23(b) shows the pattern when the sidelobe variation is restricted to 6.26 dB while achieving the required nulls. K is equal to 18 in this case. Fig.5.23(c) shows the resulting pattern when all the 30-elements are perturbed and the sidelobe variation is restricted to 0.93 dB. Fig.5.23(d) shows the pattern when the sidelobe variation is restricted to 5.42 dB while achieving the required nulls. K is equal to

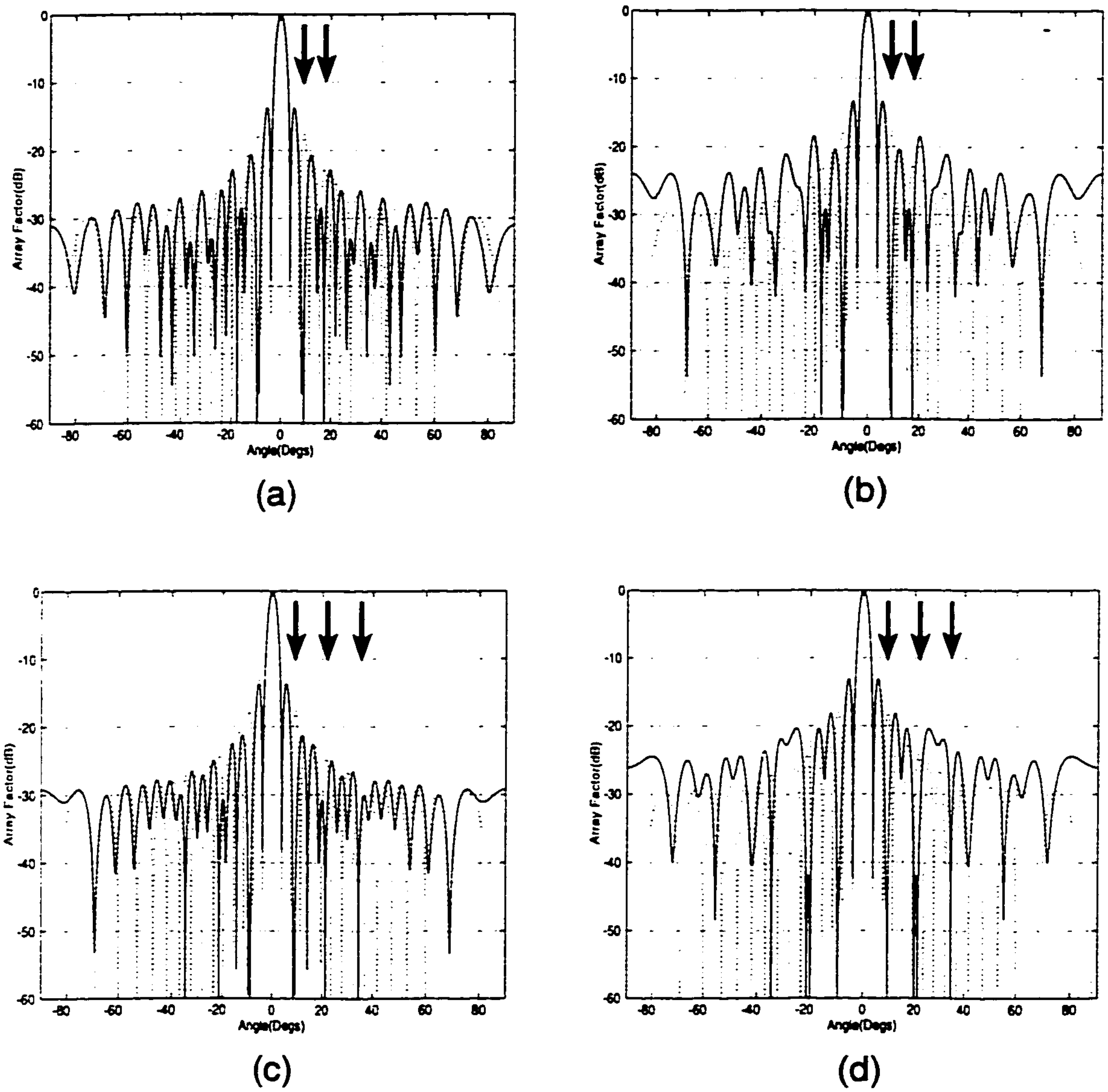


Figure 5.23: Array patterns of 30 element uniform array with (a) two nulls imposed on the peaks of second and fourth sidelobes at 9.4° and 17.4° respectively, when all elements are controlled and sidelobes restricted to 1.52 dB (b) Minimum (optimum) number of elements are controlled (K=18) with sidelobes restricted to 6.26 dB (c) three nulls imposed on the peaks of second, fifth and eighth sidelobes at 9.4°, 21.4° and 34.5° respectively, when all elements are controlled and sidelobes restricted to 0.93 dB (d) Minimum (optimum) number of elements are controlled (K=21) with sidelobes restricted to 5.42 dB

21 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.23 are given in table 5.45 and table 5.46 respectively.

The results of Fig.5.24 (a) and (b) show four nulls, which has been steered to the peaks of the second, seventh, eleventh and fourteenth sidelobes at 9.4°,30°,50° and 75.2° respectively. The results of Fig.5.24 (c) and (d) show five nulls, which has been steered to the peaks of the second, fifth, seventh, eleventh and fourteenth sidelobes at 9.4°, 21.4°, 30°, 50° and 75.2° respectively. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.24(a) when all the 30-elements are perturbed and the sidelobe variation is restricted to 1.69 dB. Fig.5.24(b) shows the pattern when the sidelobe variation is restricted to 7.86 dB while achieving the required nulls. K is equal to 23 in this case. Fig.5.24(c) shows the resulting pattern when all the 30-elements are perturbed and the sidelobe variation is restricted to 1.98 dB. Fig.5.24(d) shows the pattern when the sidelobe variation is restricted to 8.10 dB while achieving the required nulls. K is equal to 24 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.24 are given in table 5.47 and table 5.48 respectively.

The results of Fig.5.25 show six nulls, which has been steered to the peaks of the second, fourth, sixth, ninth, eleventh and fourteenth sidelobe levels at 9.4°, 17.4°, 25.6°, 39.3°, 50° and 75.2° respectively. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.25(a) when all the 30-elements are perturbed without restricting the sidelobe variation. Fig.5.25(b) shows the perturbed

ELEMENT Number	Fig. 5.23(a) (Nulls at 9.4°, 17.4°) Full(WSR) Δ_n	Fig. 5.23(b) (Nulls at 9.4°, 17.4°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.23(c) (Nulls at 9.4°, 21.4°,34.5°) Full(WSR) Δ_n	Fig. 5.23(d) (Nulls at 9.4°, 21.4°,34.5°) Partial(WSR) $\hat{\Delta}_n$
1	-0.4336	-0.3854	-0.4365	-0.3450
2	-0.2250	-0.1289	-0.2089	-0.2221
3	-0.0444	0	-0.0138	0.1175
4	0.1009	0.2026	0.1157	0.0332
5	0.2200	0.3129	0.1647	0.1941
6	0.2046	0.2325	0.2081	0.2960
7	0.1538	0.1631	0.2026	0.1928
8	0.0968	0	0.1347	0.1842
9	0.0562	0	0.0132	0
10	-0.0142	0	-0.0742	0
11	-0.0804	0	-0.1431	-0.2961
12	-0.1745	-0.1753	-0.1854	-0.4435
13	-0.2179	-0.3463	-0.2569	-0.4049
14	-0.1796	-0.3653	-0.1051	-0.0057
15	-0.0432	-0.1266	-0.0120	0
16	0.0780	0	0.0437	0
17	0.2353	0.2177	0.1183	0.1834
18	0.2820	0.2405	0.1532	0.2824
19	0.1517	0.1672	0.2403	0.2992
20	0.0926	0	0.1584	0
21	0.0233	0	0.0742	0
22	-0.0233	0	-0.0510	0
23	-0.0840	0	-0.1103	0
24	-0.1329	-0.2136	-0.2009	-0.2439
25	-0.1852	-0.2391	-0.2243	-0.2234
26	-0.1735	-0.3350	-0.2234	-0.1338
27	-0.1561	0	-0.1451	-0.0560
28	0.0353	0	0.0207	-0.0394
29	0.2671	0.1305	0.2415	0
30	0.4994	0.3619	0.4161	0.2718

Table 5.45: Computed element position perturbations for Fig. 5.23 as a function of λ.

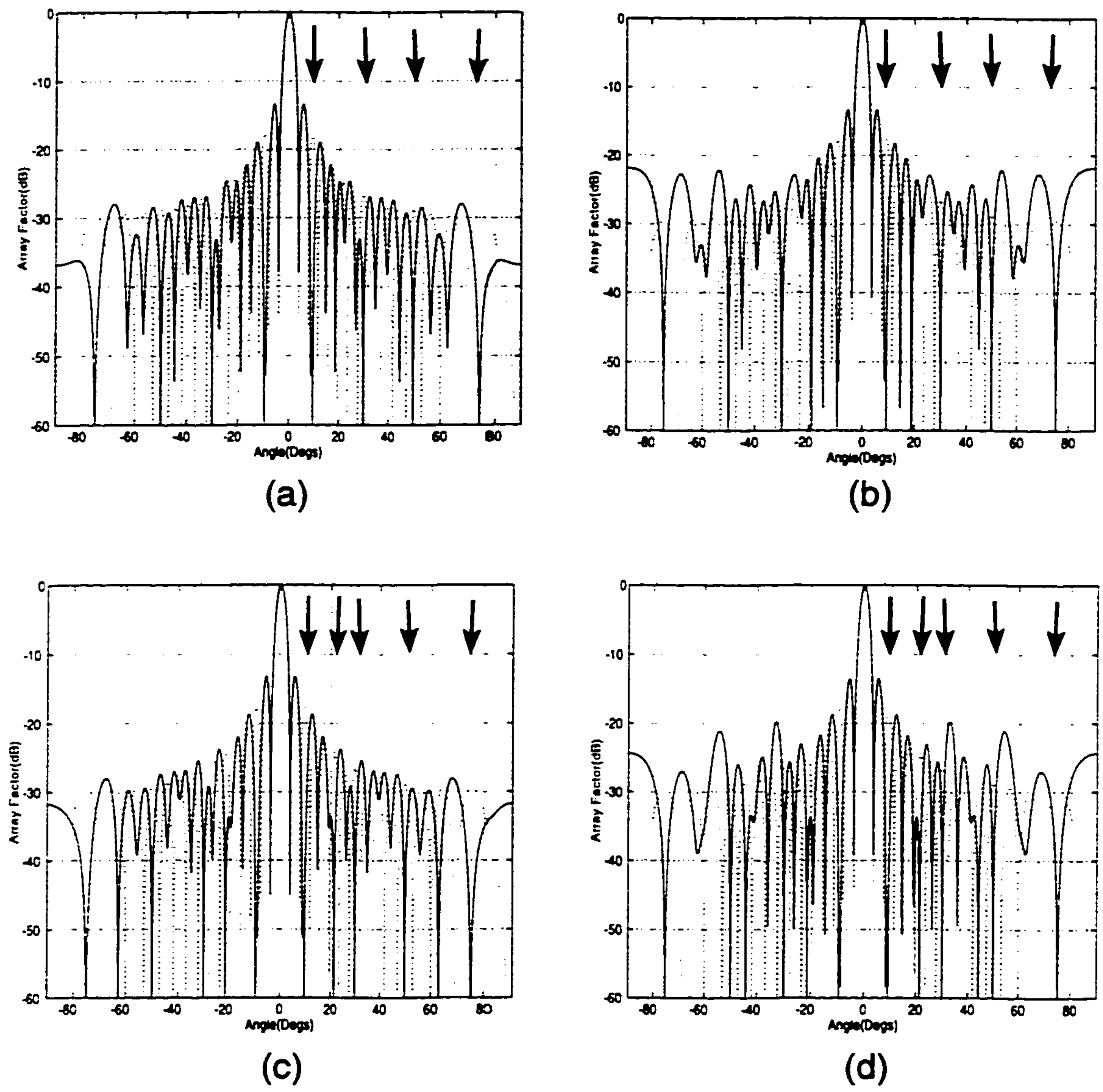


Figure 5.24: Array patterns of 30 element uniform array with (a) four nulls imposed at 9.4°,30°,50° and 75.2° respectively, when all elements are controlled and sidelobes restricted (b) Minimum (optimum) number of elements are controlled (K=23) with sidelobes restricted to 7.68 dB (c) five nulls imposed at 9.4°, 21.4°, 30°, 50° and 75.2° respectively, when all elements are controlled and sidelobes restricted (d) Minimum (optimum) number of elements are controlled (K=24) with sidelobes restricted to 8.10 dB

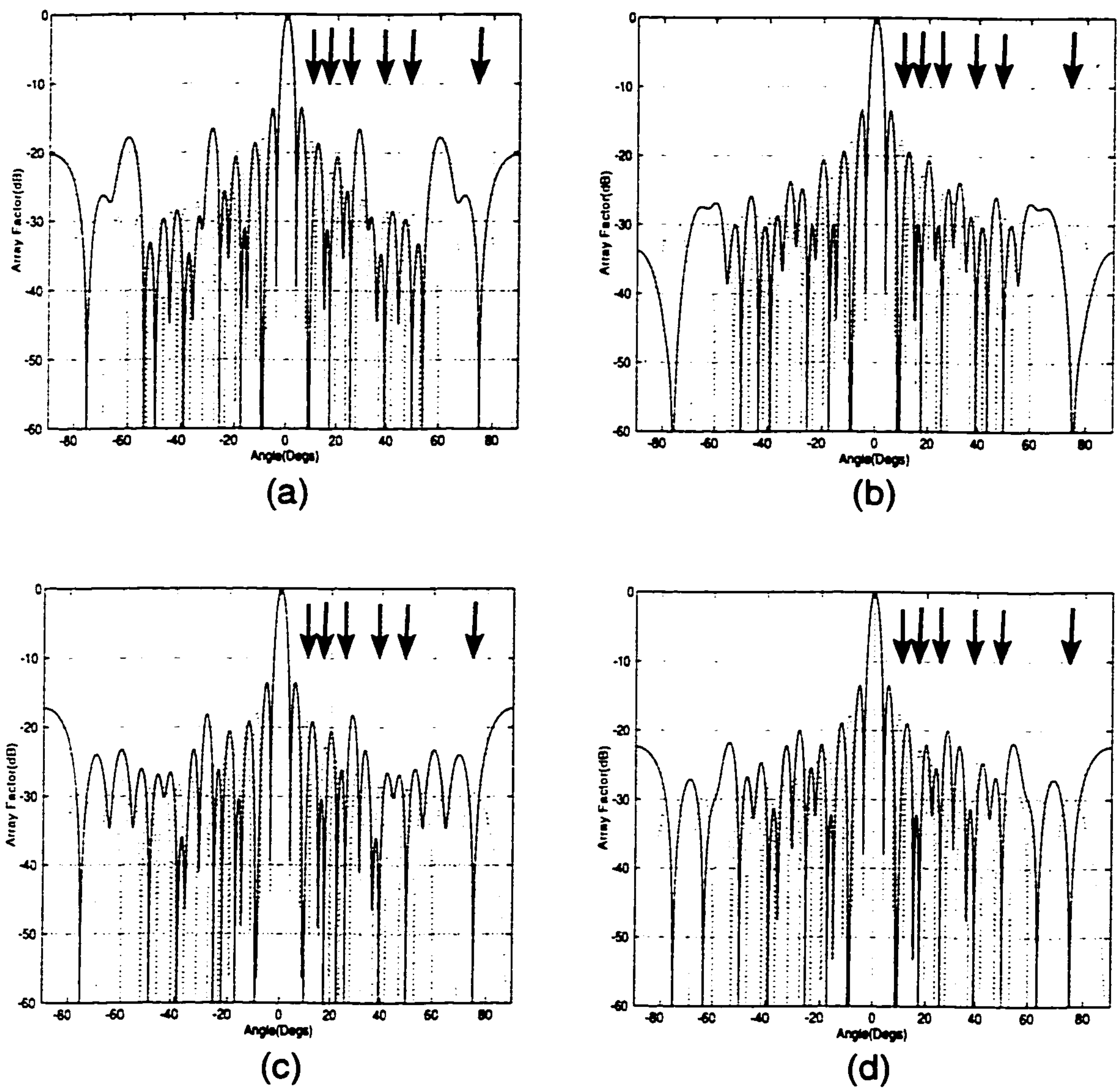


Figure 5.25: Array patterns of 30 element uniform array with six nulls imposed at 9.4°, 17.4°, 25.6°, 39.3°, 50° and 75.2° (peak of 2^{nd}, 4^{th}, 6^{th}, 9^{th}, 11^{th} and 14^{th} sidelobes). (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 3.70 dB (c)Minimum (optimum) number of elements are controlled (K=24) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=26) with sidelobes restricted to 7.45 dB

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.23(a) (Nulls at 9.4°, 17.4°) Full(WSR)	Fig. 5.23(b) (Nulls at 9.4°, 17.4°) Partial(WSR)	Fig. 5.23(c) (Nulls at 9.4°, 21.4°,34.5°) Full(WSR)	Fig. 5.23(d) (Nulls at 9.4°, 21.4°,34.5°) Partial(WSR)
Min. Controlled Elements K		30	18	30	21
SLV (dB)		1.5221	6.2640	0.9336	5.4224
DIRECTIVITY	30	30.8871	29.9186	30.8874	30.0070
HPBW (DEG.)	3.3860	3.3602	3.3848	3.3688	3.3730
SLL (dB)	-13.23	-13.8425	-13.4140	-13.7020	-13.0821
Null Depth(dB)	-17.73	-61.11	-60.03	-60.00	-60.00
No. of Generations		827	417	1067	793
CPU time (Sec)		3308	1668	4268	3172

Table 5.46: Computed Array Parameters for Fig. 5.23

pattern compared to the initial, when all the 30-elements are perturbed and the sidelobe variation is restricted to 3.70 dB. Fig.5.25(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=24 in this case. The required null has been achieved precisely, but the sidelobe level variation has changed to more than 12 dB. Fig.5.25(d) shows the pattern when the sidelobe variation is restricted to 7.45 dB while achieving the required null. K is equal to 26 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.25 are given in table 5.49 and table 5.50 respectively.

For the case of multiple nulls in 30-element uniform partially arrays with sidelobe restriction, the same behavior obtained in case of 8 and 16 element arrays is noticed here. It is observed that as we increase the number of nulls from one to six, the

ELEMENT Number	Fig. 5.24(a) (Nulls at 9.4°, 30°,50°,75.2°) Full(WSR) Δ_n	Fig. 5.24(b) (Nulls at 9.4°, 30°,50°,75.2°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.24(c) (Nulls at 9.4°,21.4°, 30°,50°,75.2°) Full(WSR) Δ_n	Fig. 5.24(d) (Nulls at 9.4°,21.4°, 30°,50°,75.2°) Partial(WSR) $\hat{\Delta}_n$
1	-0.3157	-0.1946	-0.3265	-0.2782
2	-0.1779	-0.2715	-0.2386	-0.2058
3	-0.0240	-0.0961	-0.0591	0
4	0.1038	0.1738	0.0425	0.1330
5	0.2203	0.2914	0.1682	0.1329
6	0.2771	0.2539	0.2518	0.2101
7	0.2331	0.1981	0.2131	0.2746
8	0.1603	0.1833	0.0963	0.1921
9	0.0415	0.0131	0.0484	-0.0013
10	-0.0469	0	-0.1040	0
11	-0.1579	-0.1754	-0.1822	-0.1402
12	-0.2456	-0.2512	-0.3035	-0.2520
13	-0.2565	-0.2900	-0.2698	-0.2681
14	-0.1700	-0.1659	-0.1718	-0.1426
15	-0.0749	-0.0085	-0.0194	0
16	0.0348	0	0.0595	0.0051
17	0.2172	0.2081	0.1601	0.1573
18	0.2419	0.2580	0.3061	0.2506
19	0.2210	0.1774	0.2737	0.2525
20	0.1051	0.1566	0.1739	0.1771
21	0.0507	0	0.0609	0
22	-0.0545	0	-0.0339	0
23	-0.0818	-0.2133	-0.1681	-0.1872
24	-0.1746	-0.2302	-0.1797	-0.2637
25	-0.2295	-0.3178	-0.1382	-0.2427
26	-0.2197	-0.2012	-0.1194	-0.1579
27	-0.0720	0	-0.0872	-0.1204
28	0.0540	0	-0.0005	0
29	0.1874	0	0.1557	0.2549
30	0.2735	0.2537	0.3242	0.2762

Table 5.47: Computed element position perturbations for Fig. 5.24 as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.24(a) (Nulls at 9.4°, 30°,50°,75.2°) Full(WSR)	Fig. 5.24(b) (Nulls at 9.4°, 30°,50°,75.2°) Partial(WSR)	Fig. 5.24(c) (Nulls at 9.4°,21.4°, 30°,50°,75.2°) Full(WSR)	Fig. 5.24(d) (Nulls at 9.4°,21.4°, 30°,50°,75.2°) Partial(WSR)
Min. Controlled Elements K		30	23	30	24
SLV (dB)		1.6909	7.6834	1.9868	8.1022
DIRECTIVITY	30	30.3006	30.0031	30.3885	29.4966
HPBW (DEG.)	3.3860	3.3813	3.3966	3.3636	3.3839
SLL (dB)	-13.23	-13.4178	-13.3685	-13.2123	-13.5118
Null Depth(dB)	-17.73	-60.00	-60.12	-60.50	-60.33
No. of Generations		2173	1037	833	502
CPU time (Sec)		8692	4148	3332	2008

Table 5.48: Computed Array Parameters for Fig. 5.24

minimum no of controlled elements K and SLV has also increased. From the tables 5.46. 5.48 and 5.50 comparing the array parameters of partially adaptive array, with the corresponding fully adaptive array, when multiple nulls are imposed, one notices that directivity, HPBW, SLL are almost unchanged. The sidelobe variation (SLV) in case of partially adaptive arrays is slightly higher than in fully adaptive case, since it is obvious that the best array parameters are obtained, when all the 30-elements are perturbed, as it affords the greatest control over the array response.

5.6.3 Determination of the realizable minimum number of controlled elements

From the results obtained, it is found that the minimum number K and the location of the controlled elements in partially adaptive arrays depend on the number and

ELEMENT Number	Fig. 5.25(a) Full(WOSR) Δ_n	Fig. 5.25(b) Full(WSR) Δ_n	Fig. 5.25(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.25(d) Partial(WSR) $\hat{\Delta}_n$
1	-0.3190	-0.3198	-0.3048	-0.3050
2	-0.2438	-0.2240	-0.2986	-0.3051
3	-0.0562	-0.0294	0	0
4	0.1886	0.1299	0.2305	0.1842
5	0.1700	0.2066	0.1797	0.1745
6	0.1739	0.2039	0.1970	0.1294
7	0.1814	0.1723	0.2010	0.1650
8	0.1939	0.0645	0.2370	0.1742
9	0.0697	0.0464	0	0.0325
10	-0.0317	-0.0512	0	0
11	-0.1801	-0.0466	-0.1023	-0.0977
12	-0.1147	-0.2140	-0.0722	-0.1288
13	-0.2202	-0.2813	-0.2375	-0.2431
14	-0.2377	-0.2850	-0.2549	-0.2993
15	-0.2206	-0.1894	-0.0806	-0.0479
16	0.2541	0.1119	0.2562	0.2070
17	0.3245	0.2967	0.3157	0.3212
18	0.2141	0.2074	0.2247	0.2121
19	0.1062	0.1491	0.1351	0.1881
20	0.0927	0.0536	0.1421	0.1292
21	0.0888	0.0622	0	0
22	-0.1439	-0.0157	0	0
23	-0.1828	-0.0758	-0.2071	-0.2106
24	-0.1639	-0.2035	-0.1440	-0.2040
25	-0.1478	-0.1659	-0.2003	-0.2087
26	-0.1792	-0.1656	-0.1594	-0.1686
27	-0.2838	-0.1058	-0.2617	-0.1891
28	0.0149	0.0183	0	-0.0676
29	0.2377	0.2608	0.2044	0.1962
30	0.2491	0.4188	0.2592	0.2983

Table 5.49: Computed element position perturbations for Fig. 5.25 when six nulls are steered at 9.4°,17.4°,25.6°,39.3°,50° and 75.2°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.25(a) Full(WOSR)	Fig. 5.25(b) Full(WSR)	Fig. 5.25(c) Partial(WOSR)	Fig. 5.25(d) Partial(WSR)
Min. Controlled Elements K		30	30	24	26
SLV (dB)		11.6496	3.7045	12.3249	7.4534
DIRECTIVITY	30	28.8142	30.5036	29.3534	29.7033
HPBW (DEG.)	3.3860	3.3850	3.3659	3.3894	3.3835
SLL (dB)	-13.23	-13.5569	-13.5606	-13.5677	-13.5924
Null Depth(dB)	-17.73	-60.11	-60.06	-60.10	-60.02
No. of Generations		2691	842	1651	461
CPU time (Sec)		1230.3	3368	550.3	1844

Table 5.50: Computed Array Parameters for Fig. 5.25 when six nulls are steered at 9.4°,17.4°,25.6°,39.3°,50° and 75.2°.

location of nulls. In other words, the minimum number K and location of the controlled elements for partially adaptive method vary with the number and location of imposed nulls. This is not desirable in element position perturbation technique, since it requires to install a motor for each element and hence the number of motors are not being reduced. Therefore, this technique can only be realistically implemented if a limited number of motors are fixed at some optimum elements locations of an array, capable of providing a single or multiple imposed nulls at locations covering most of the sidelobe region, while other array parameters are nearly kept unchanged.

To implement this technique for a single null, the null location which requires the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steer-

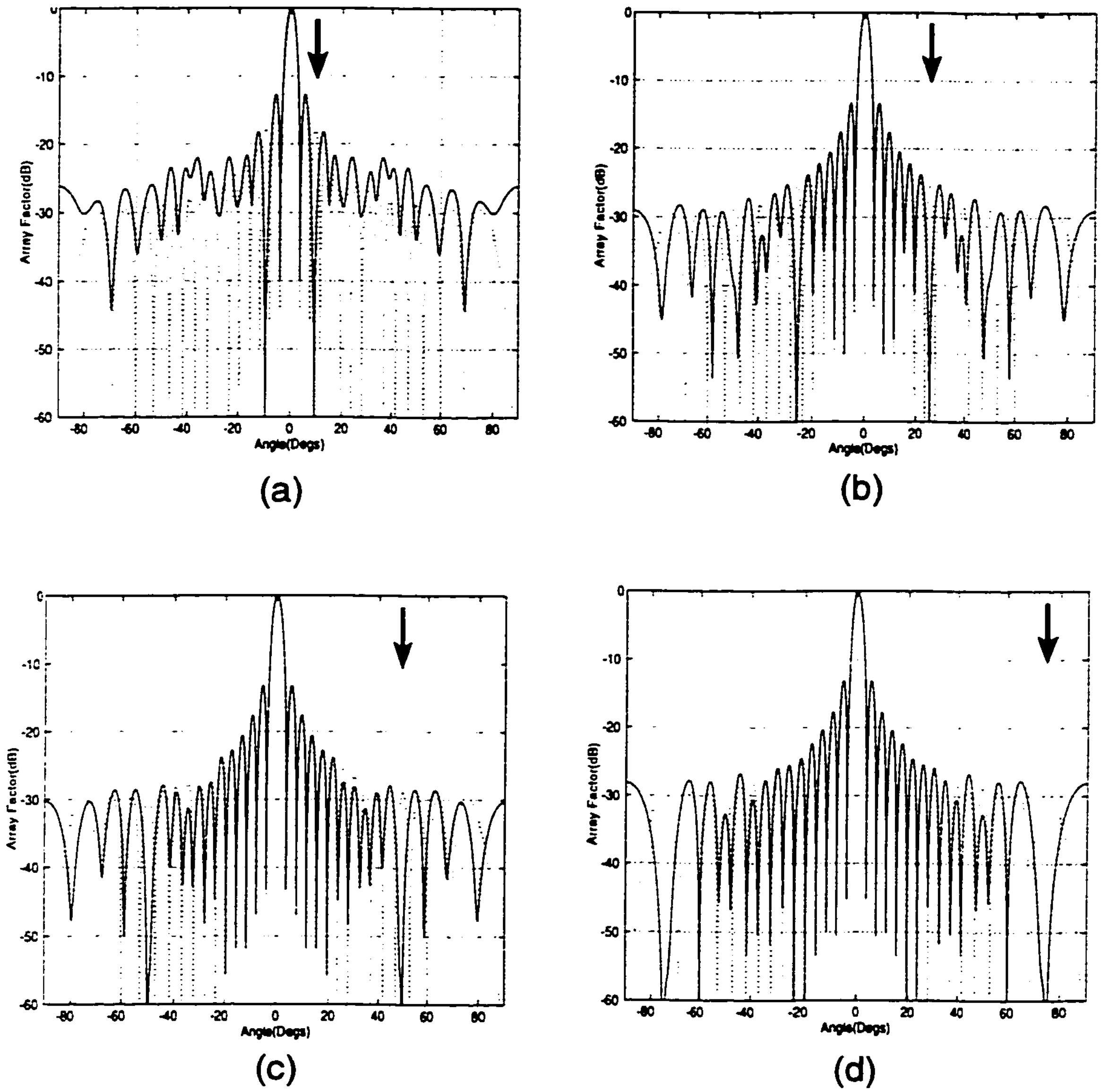


Figure 5.26: Array patterns of 30 element uniform array perturbing 16 fixed elements with (a) one null imposed on the peak of second sidelobe at 9.4° (b) one null imposed on the peak of sixth sidelobe at 25.6° (c) one null imposed on the peak of eleventh sidelobe at 50° (d) one null imposed on the peak of fourteenth sidelobe at 75.6°

ing. In the case of 30-element uniform partially adaptive arrays a null on the peak of second sidelobe requires the use of 16 controlled elements. Now keeping the location of these 16 controlled elements fixed, a single null is imposed on the peak of sixth sidelobe as shown in Fig.5.26(b) with sidelobes restricted to 1.32 dB. A single null is imposed on the peak of eleventh sidelobe as shown in Fig.5.26(c) with sidelobes restricted to 0.83 dB and a single null is imposed on the peak of fourteenth sidelobe as shown in Fig.5.26(d) with sidelobes restricted to 1.81 dB. Therefore perturbing only these 16 fixed elements out of 30-elements, we can steer a single null any where in the sidelobe region of an 30-element uniform partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.26 are given in table 5.51 and table 5.52 respectively.

The results given in table 5.52 show that the sidelobe variation (SLV) when the null is imposed at sixth, eleventh or fourteenth sidelobe, is higher than the previous results inspite of increasing the no. of controlled elements. This shows that, for a particular null location the SLV depends on the location of the minimum controlled elements.

To implement this technique for two nulls, the null locations which requires the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steering. In the case of 30-element uniform partially adaptive arrays two nulls on the peak of second and fourth sidelobe requires the use of 18 controlled elements. Now

ELEMENT Number	Fig. 5.26(a) (Null at 9.4°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.26(b) (Null at 25.6°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.26(c) (Null at 50°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.26(d) (Null at 75.6°) Partial(WSR) $\hat{\Delta}_n$
1	-0.3541	-0.0688	-0.0157	0.0005
2	-0.1115	0.0326	0.0401	0.0179
11	-0.1545	0.0431	-0.0131	-0.0278
12	-0.3260	0.0706	0.0403	0.0165
13	-0.3222	0.0244	0.0003	-0.0265
17	0.2638	0.0405	0.0210	-0.0094
18	0.4179	0.0147	0.0007	0.0208
19	0.4288	-0.0695	-0.0177	-0.0114
20	0.2902	0.0046	0.0117	0.0295
21	0.1904	0.0507	-0.0228	-0.0031
23	-0.2193	-0.0581	0.0279	-0.0170
24	-0.2093	-0.0980	-0.0329	0.0150
25	-0.3200	0.0468	-0.0078	-0.0008
26	-0.2935	0.0949	0.0133	0.0039
29	0.2098	-0.0389	-0.0475	-0.0136
30	0.4008	0.0235	-0.0009	-0.0248

Table 5.51: Computed element position perturbations for Fig. 5.26 as a function of λ.

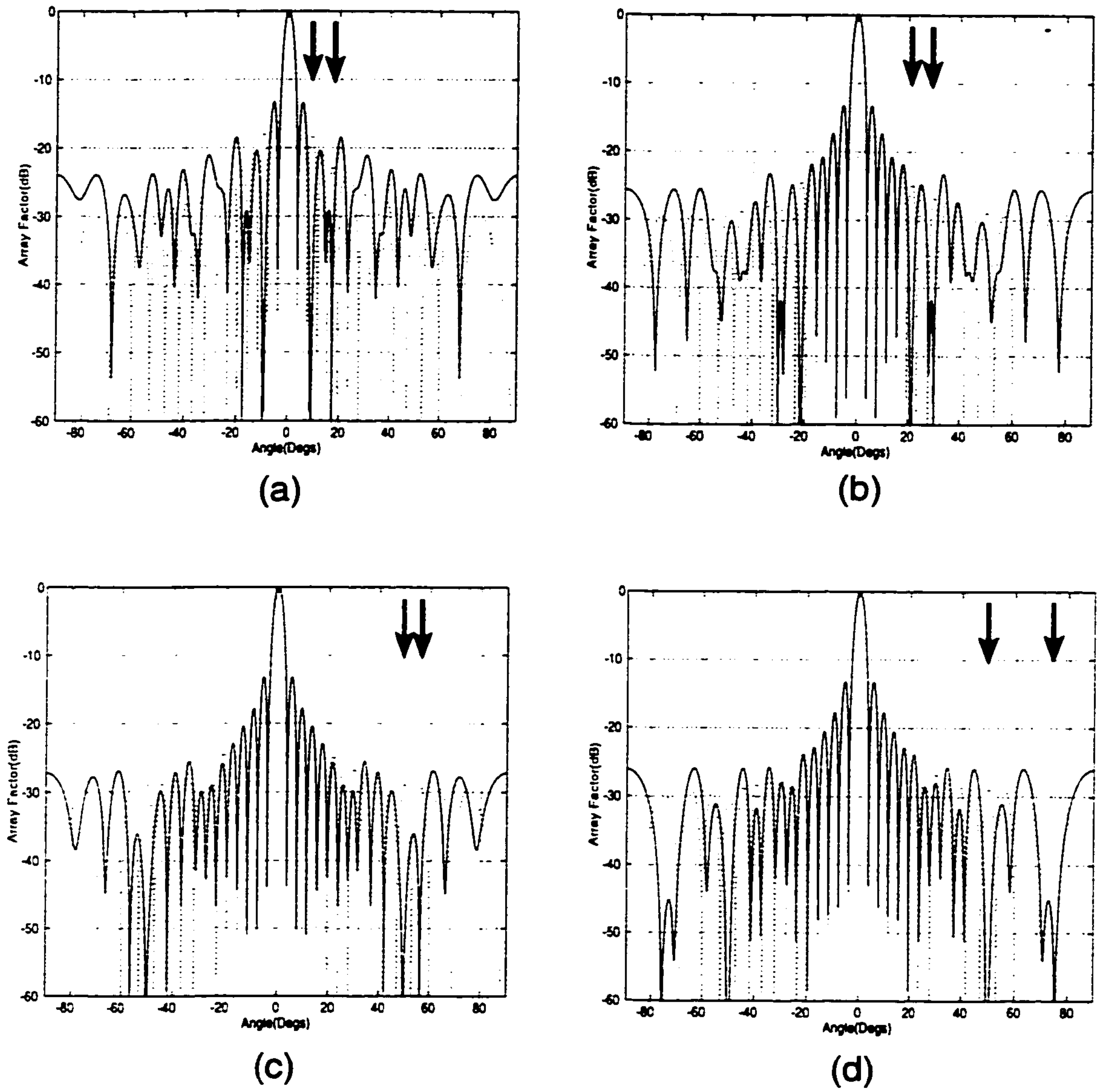


Figure 5.27: Array patterns of 30 element uniform array perturbing 18 fixed elements with (a) two nulls imposed on the peak of second and fourth sidelobes at 9.4° and 17.4° (b) two nulls imposed on the peak of fifth and seventh sidelobes at 21.4° and 30° (c) two nulls imposed on the peak of eleventh and twelveth sidelobes at 50° and 56° (d) two nulls imposed on the peak of eleventh and fourteenth sidelobes at 50° and 75.2°

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.26(a) (Null at 9.4°) Partial (WSR)	Fig. 5.26(b) (Null at 25.6°) Partial (WSR)	Fig. 5.26(c) (Null at 50°) Partial (WSR)	Fig. 5.26(d) (Null at 75.6°) Partial (WSR)
No. of Controlled Elements		16	16	16	16
SLV (dB)		6.0276	1.3223	0.8300	1.8190
DIRECTIVITY	30	30.6171	30.1793	30.0638	29.9450
HPBW (DEG.)	3.3860	3.3441	3.3860	3.3894	3.3872
SLL (dB)	-13.23	-12.7403	-13.4090	-13.2175	-13.1509
Initial Null Depth(dB)		-17.73	-25.51	-28.94	-29.53
Final Null Depth(dB)		-67.19	-67.07	-63.97	-80.57
No. of Generations		501	108	62	43
CPU time (Sec)		2004	432	248	172

Table 5.52: Computed Array Parameters for Fig. 5.26

keeping the location of these 18 controlled elements fixed, two nulls are imposed on the peak of fifth and seventh sidelobes as shown in Fig.5.27(b) with sidelobes restricted to 4.12 dB . Two nulls are imposed on the peak of eleventh and twelveth sidelobes as shown in Fig.5.27(c) with sidelobes restricted to 2.58 dB and two nulls are imposed on the peak of eleventh and fourteenth sidelobe as shown in Fig.5.27(d) with sidelobes restricted to 3.52 dB. Therefore perturbing only these 18 fixed elements out of 30-elements, we can steer two nulls any where in the sidelobe region of an 30-element uniform partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.27 are given in table 5.53 and table 5.54 respectively.

To implement this technique for three nulls, the null locations which requires the highest no. of controlled elements is chosen. Using these controlled elements,

ELEMENT Number	Fig. 5.27(a) (Nulls at 9.4°,17.4°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.27(b) (Nulls at 21.1°,30°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.27(c) (Nulls at 50°,56°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.27(d) (Nulls at 50°,75.2°) Partial(WSR) $\hat{\Delta}_n$
1	-0.3854	-0.0735	-0.0339	0.0034
2	-0.1289	0.0169	0.0394	0.0238
4	0.2026	0.0171	0.0653	0.0237
5	0.3129	-0.0834	-0.0323	0.0000
6	0.2325	-0.0824	-0.0266	-0.0045
7	0.1631	0.0015	0.0266	0.0106
12	-0.1753	-0.0940	0.0000	0.0556
13	-0.3463	0.0234	-0.0131	-0.0338
14	-0.3653	0.1222	0.0000	-0.0056
15	-0.1266	0.0262	0.0018	0.0017
17	0.2177	-0.1126	-0.0025	-0.0167
18	0.2405	0.0000	0.0244	0.0384
19	0.1672	0.0585	-0.0011	-0.0432
24	-0.2136	0.0227	-0.0483	-0.0410
25	-0.2391	0.0896	0.0000	0.0018
26	-0.3350	0.0719	0.0410	0.0244
29	0.1305	-0.0281	-0.0380	-0.0318
30	0.3619	0.0963	0.0225	0.0173

Table 5.53: Computed element position perturbations for Fig. 5.27 as a function of λ.

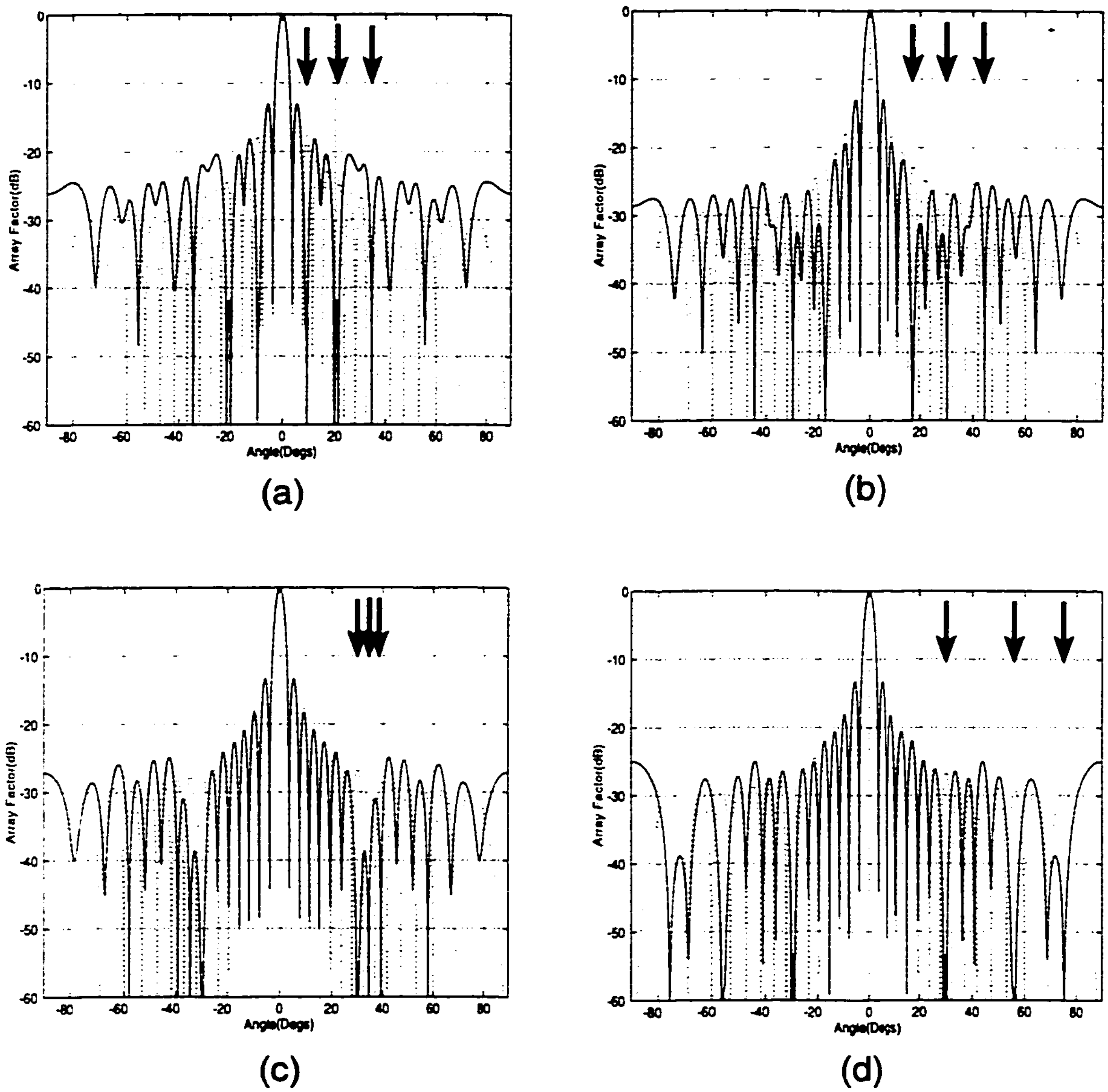


Figure 5.28: Array patterns of 30 element uniform array perturbing 21 fixed elements with (a) three nulls imposed on the peaks of second, fifth and eighth sidelobes at 9.4°, 21.4° and 34.5° (b) three nulls imposed on the peaks of fourth, seventh and tenth sidelobes at 17.4°, 30° and 44.4°(c) three nulls imposed on the peaks of seventh, eighth and ninth sidelobes at 30°, 34.5° and 39.3°(d) three nulls imposed on the peaks of seventh, twelveth and fourteenth sidelobes at 30°, 56.4° and 75.2°

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.27(a) (Nulls at 9.4°, 17.4°) Partial (WSR)	Fig. 5.27(b) (Nulls at 21.1°, 30°) Partial (WSR)	Fig. 5.27(c) (Nulls at 50°, 56°) Partial (WSR)	Fig. 5.27(d) (Nulls at 50°, 75.2°) Partial (WSR)
No. of Controlled Elements		18	18	18	18
SLV (dB)		6.2640	4.1218	2.5837	3.5253
DIRECTIVITY	30	29.9186	29.9661	30.1089	29.9704
HPBW (DEG.)	3.3860	3.3848	3.3703	3.3876	3.3895
SLL (dB)	-13.23	-13.4140	-13.2698	-13.2199	-13.2546
Initial Null Depth(dB)		-17.73	-24.25	-28.94	-28.94
Final Null Depth(dB)		-60.03	-60.29	-60.00	-60.10
No. of Generations		417	309	125	504
CPU time (Sec)		1668	309	125	504

Table 5.54: Computed Array Parameters for Fig. 5.27

nulls are imposed at different locations to test that the system is functional for null steering. In the case of 30-element uniform partially adaptive arrays three nulls on the peaks of second, fifth and eighth sidelobes requires the use of 21 controlled elements. Now keeping the location of these 21 controlled elements fixed, three nulls are imposed on the peaks of fourth, seventh and tenth sidelobes as shown in Fig.5.28(b) with sidelobes restricted to 3.40 dB . Three nulls are imposed on the peaks of seventh, eighth and ninth sidelobes as shown in Fig.5.28(c) with sidelobes restricted to 3.71 dB and three nulls are imposed on the peaks of seventh, twelveth and fourteenth sidelobes as shown in Fig.5.28(d) with sidelobes restricted to 4.61 dB. Therefore perturbing only these 21 fixed elements out of 30-elements, we can steer three nulls any where in the sidelobe region of an 30-element uniform partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.28 are given in table 5.55 and table 5.56 respectively.

The 30-element uniform partially adaptive array is not suitable for steering four or

ELEMENT Number	Fig. 5.28(a) (Nulls at 9.4°, 21.4°,34.5°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.28(b) (Nulls at 17.4°, 30°,44.4°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.28(c) (Nulls at 30°, 34.5°,39.3°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.28(d) (Nulls at 30°, 56.4°,75.2°) Partial(WSR) $\hat{\Delta}_n$
1	-0.3450	-0.2178	-0.0905	-0.0297
2	-0.2221	0.0753	0.0579	0.0175
3	-0.1175	0.1448	0.0158	0.0143
4	0.0332	0.0583	-0.0488	-0.0042
5	0.1941	0.0552	0.0000	-0.0385
6	0.2960	-0.0480	0.0509	0.0509
7	0.1928	-0.0742	0.0023	0.0226
8	0.1842	-0.0640	-0.0122	-0.0160
11	-0.2961	0.0000	0.0000	0.0000
12	-0.4435	-0.0415	-0.0086	-0.0194
13	-0.4049	-0.0971	-0.0199	-0.0647
14	-0.0057	-0.0737	0.0087	0.0293
17	0.1834	0.0512	-0.0114	-0.0465
18	0.2824	0.1311	0.0320	0.0247
19	0.2992	0.0878	0.0081	0.0205
24	-0.2439	0.0604	-0.0022	-0.0402
25	-0.2234	0.0249	-0.0733	-0.0468
26	-0.1338	-0.0406	-0.0099	0.0402
27	-0.0560	-0.0813	0.0784	0.0224
28	-0.0394	-0.0801	-0.0139	-0.0035
30	0.2718	0.1930	0.0757	0.0506

Table 5.55: Computed element position perturbations for Fig. 5.28 as a function of λ.

more nulls, because steering four nulls requires a minimum of 23 controlled elements out of 30 elements.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.28(a) (Nulls at 9.4°, 21.4°,34.5°) Partial (WSR)	Fig. 5.28(b) (Nulls at 17.4°, 30°,44.4°) Partial (WSR)	Fig. 5.28(c) (Nulls at 30°, 34.5°,39.3°) Partial (WSR)	Fig. 5.28(d) (Nulls at 30°, 56.4°,75.2°) Partial (WSR)
No. of Controlled Elements		21	21	21	21
SLV (dB)		5.4224	3.4075	3.7115	4.6100
DIRECTIVITY	30	30.0070	30.3601	30.1502	30.0165
HPBW (DEG.)	3.3860	3.3730	3.3768	3.3818	3.3841
SLL (dB)	-13.23	-13.0821	-12.9920	-13.3320	-13.2703
Initial Null Depth(dB)		-17.73	-22.66	-26.53	-26.53
Final Null Depth(dB)		-60.00	-60.24	-60.55	-60.94
No. of Generations		793	337	194	105
CPU time (Sec)		3172	1348	776	420

Table 5.56: Computed Array Parameters for Fig. 5.28

5.7 Simulation results of 30 element chebyshev partially adaptive arrays

5.7.1 Introduction

In this section, we study the array performance and behavior of its parameters such as half-power beam width (HPBW), directivity , sidelobe level (SLL), the minimum number of controlled elements K and the sidelobe variation (SLV) on 30-element chebyshev array with sidelobe level of -30dB and with element spacing of 0.5 λ. The validity of the proposed partially adaptive method is examined by first placing a single null and then placing multiple nulls on the peaks of sidelobes. The results are compared with the fully adaptive case with and without sidelobe restrictions.

5.7.2 Simulation results

The results of Fig.5.29 show one null, which has been steered to the peak of the second sidelobe level at 9.5°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.29(a) when all the 30-elements are perturbed without restricting the sidelobe variation. Fig.5.29(b) shows the perturbed pattern compared to the initial, when all the 30-elements are perturbed and the sidelobe variation is restricted to 1.06 dB. Fig.5.29(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=6 in this case. The required null has been achieved precisely, but the sidelobe level variation has changed to more than 10 dB. Fig.5.29(d) shows the pattern when the sidelobe variation is restricted to 2.50 dB while achieving the required null. K is equal to 10 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.29 are given in table 5.57 and table 5.58 respectively.

The results of Fig.5.30 (a) and (b) show one null, which has been steered to the peak of the sixth sidelobe level at 25° and the results of Fig.5.30 (c) and (d) show one null, which has been steered to the peak of the twelveth sidelobe level at 56°. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.30(a) when all the 30-elements are perturbed and the sidelobe variation is restricted to 0.54 dB. Fig.5.30(b) shows the pattern when the sidelobe variation is restricted to 1.09 dB while achieving the required null. K is equal to 10 in this case. Fig.5.30(c)

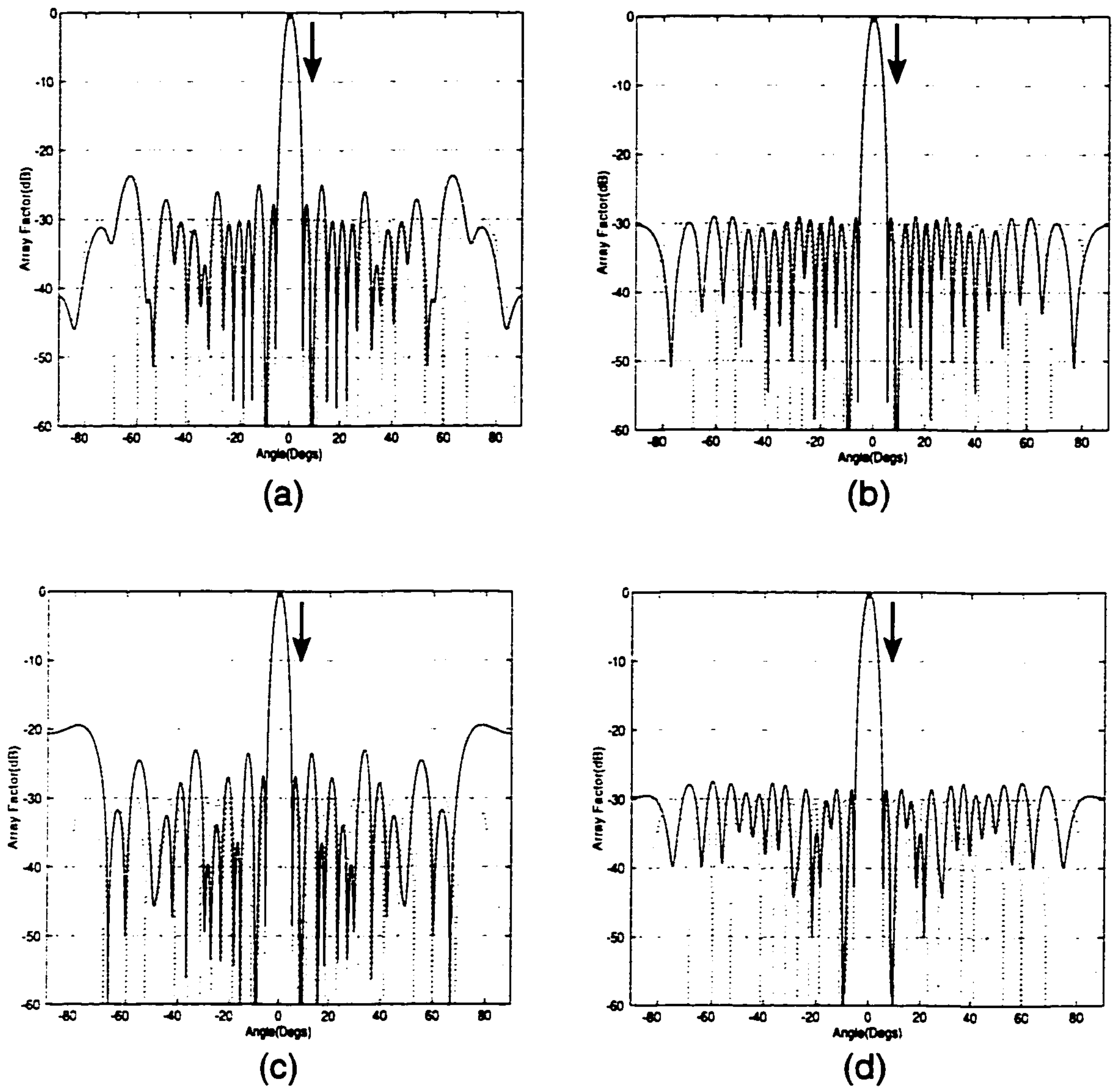


Figure 5.29: Array patterns of 30 element chebyshev array with -30 dB side lobe level and one null imposed on the peak of second sidelobe at 9.5° (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 1.06 dB (c)Minimum (optimum) number of elements are controlled (K=8) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=10) with sidelobes restricted to 2.50 dB

ELEMENT Number	Fig. 5.29(a) Full(WOSR) Δ_n	Fig. 5.29(b) Full(WSR) Δ_n	Fig. 5.29(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.29(d) Partial(WSR) $\hat{\Delta}_n$
1	-0.0202	-0.1879	0	-0.1967
2	-0.0006	-0.1019	0	0
3	-0.0318	0.0009	0	0
4	0.0062	0.0944	0	0
5	0.0630	0.0864	0	0
6	0.0460	0.0600	0	0.0887
7	0.0479	0.0423	0.1183	0.0780
8	0.0504	0.0404	0	0
9	0.0677	0.0164	0	0
10	-0.0541	-0.0215	0	0
11	-0.0440	-0.0210	0	-0.0745
12	-0.0635	-0.0200	-0.0832	-0.1045
13	-0.0722	-0.0158	-0.0875	-0.1022
14	-0.0529	-0.0152	-0.1084	-0.0381
15	-0.0660	-0.0109	0	0
16	0.0263	-0.0009	0	0
17	0.0389	0.0222	0.1002	0
18	0.0493	0.0370	0.0989	0
19	0.0385	0.0337	0.0904	0.0378
20	0.0448	0.0312	0	0
21	0.0004	0.0003	0	0
22	-0.0334	-0.0064	0	0
23	-0.0519	-0.0381	0	0
24	-0.0564	-0.0868	-0.0943	0
25	-0.0316	-0.0852	0	0
26	-0.0353	-0.0803	0	-0.0499
27	0.0069	-0.0493	0	0
28	0.0345	-0.0078	0	0
29	0.0302	0.0359	0	0
30	0.0516	0.1489	0	0.2754

Table 5.57: Computed element position perturbations for Fig. 5.29 when a single null is steered at 9.5°. Perturbations are given as a function of λ.

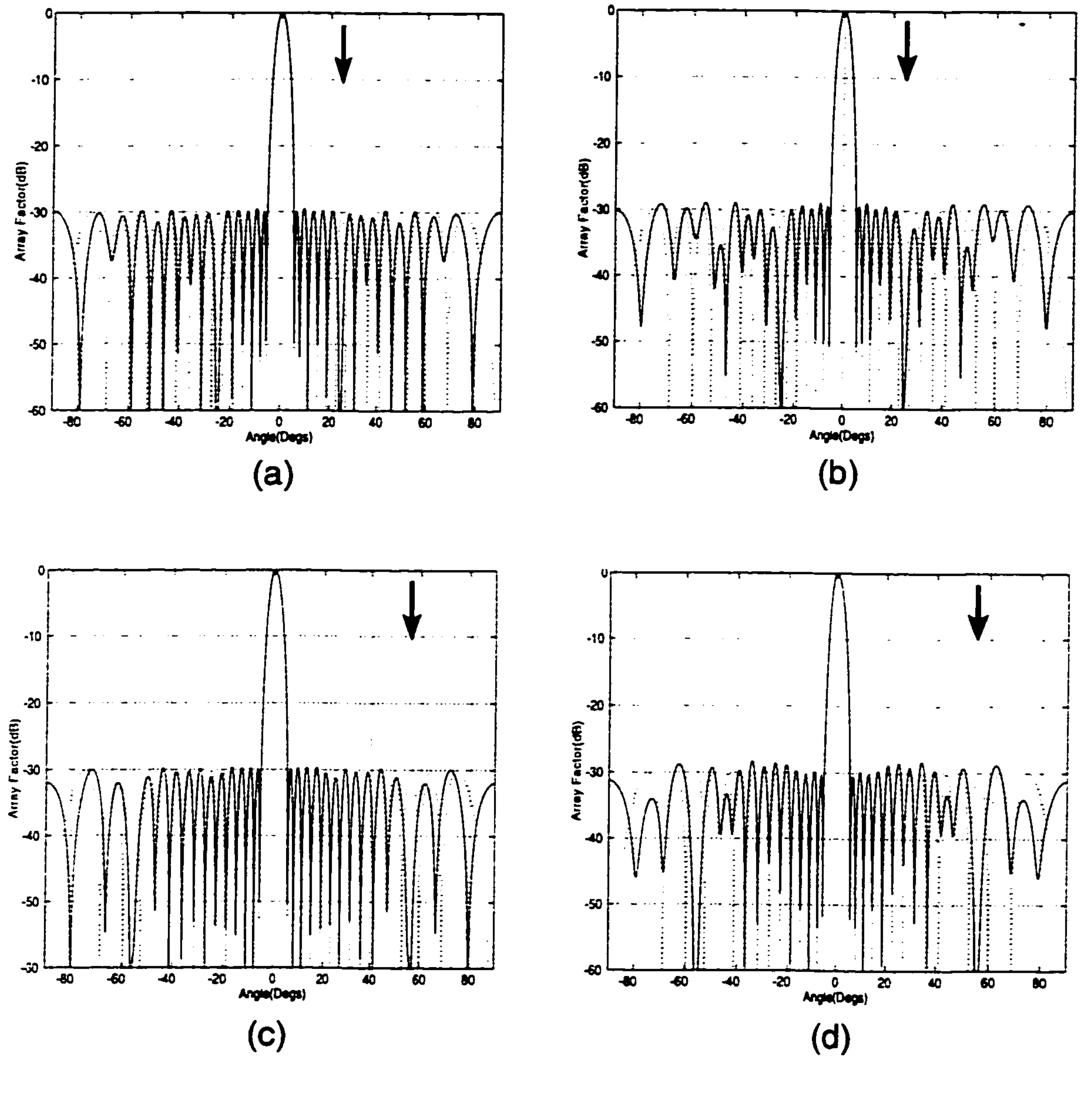


Figure 5.30: Array patterns of 30 element chebyshev array with -30 dB sidelobe level with (a) one null imposed on the peak of sixth sidelobe at 25° when all elements are controlled and sidelobes restricted to 0.54 dB (b) Minimum (optimum) number of elements are controlled (K=10) with sidelobes restricted to 1.09 dB (c) one null imposed on the peak of twelveth sidelobe at 56° when all elements are controlled and sidelobes restricted to 0.38 dB (d) Minimum (optimum) number of elements are controlled (K=6) with sidelobes restricted to 1.70 dB

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.29(a) Full(WOSR)	Fig. 5.29(b) Full(WSR)	Fig. 5.29(c) Partial(WOSR)	Fig. 5.29(d) Partial(WSR)
Min. Controlled Elements K		30	30	8	10
SLV (dB)		6.3398	1.0608	10.5289	2.5085
DIRECTIVITY	26.24	26.1272	26.2033	25.6613	26.3949
HPBW (DEG.)	4.1631	4.1548	4.1688	4.1587	4.1431
SLL (dB)	-30	-23.6602	-28.9392	-19.4711	-27.4915
Null Depth(dB)	-30	-64.37	-61.11	-63.39	-65.46
No. of Generations		243	245	130	297
CPU time (Sec)		81	980	43.3	1188

Table 5.58: Computed Array Parameters for Fig. 5.29 when a single null is steered at 9.5°.

shows the resulting pattern when all the 30-elements are perturbed and the sidelobe variation is restricted to 0.38 dB. Fig.5.30(d) shows the pattern when the sidelobe variation is restricted to 1.70 dB while achieving the required null. K is equal to 6 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.30 are given in table 5.59 and table 5.60 respectively.

In general for the case of a single null in 30-element chebyshev partially adaptive arrays with sidelobe restriction, the same behavior obtained in case of 8 and 16 element arrays is noticed here. It is observed that as we steer a single null towards the main beam, the minimum no of controlled elements K, SLL and SLV is increase, while the other array parameters such as directivity and HPBW remain almost unchanged. This is due to the reason that steering of a null near the main beam requires a higher order of degree's of freedom.

It is also observed that, when the sidelobe restriction is applied, the number of

ELEMENT Number	Fig. 5.30(a) (Null at 25°) Full(WSR) Δ_n	Fig. 5.30(b) (Null at 25°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.30(c) (Null at 56°) Full(WSR) Δ_n	Fig. 5.30(d) (Null at 56°) Partial(WSR) $\hat{\Delta}_n$
1	-0.0398	0	-0.0000	0
2	0.0935	0.1434	0.0453	0
3	-0.0003	0	-0.0351	-0.0532
4	-0.0376	0	0.0165	0.0549
5	-0.0446	-0.1018	-0.0254	0
6	0.0001	0	0.0079	0
7	0.0238	0.0425	-0.0037	0
8	0.0098	0	-0.0067	0
9	-0.0081	0	0.0004	0
10	-0.0236	0	-0.0099	0
11	0.0005	0	0.0008	0
12	0.0185	0	-0.0001	0
13	-0.0007	0	-0.0010	0
14	-0.0187	-0.0168	0.0003	0
15	-0.0058	0	-0.0072	-0.0280
16	0.0077	0	0.0038	0
17	0.0048	0.0055	-0.0051	-0.0287
18	-0.0004	0	0.0016	0
19	-0.0180	-0.0382	0.0007	0
20	-0.0007	0	-0.0009	-0.0273
21	0.0121	0.0217	0.0148	0
22	0.0343	0.0381	-0.0103	0
23	-0.0008	0	0.0006	0
24	-0.0008	0	0.0001	0
25	0.0104	0	0.0007	0
26	0.0539	0	0.0420	0
27	0.0045	0	-0.0283	0
28	-0.0192	-0.0257	0.0339	0.0577
29	-0.0737	-0.1513	-0.0430	0
30	0.0233	0	0.0004	0

Table 5.59: Computed element position perturbations for Fig. 5.30 as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.30(a) (Null at 25°) Full(WSR)	Fig. 5.30(b) (Null at 25°) Partial(WSR)	Fig. 5.30(c) (Null at 56°) Full(WSR)	Fig. 5.30(d) (Null at 56°) Partial(WSR)
Min. Controlled Elements K		30	10	30	6
SLV (dB)		0.5451	1.0950	0.3857	1.7017
DIRECTIVITY	26.2448	26.3223	26.3083	26.2954	26.2452
HPBW (DEG.)	4.1631	4.1581	4.1667	4.1612	4.1632
SLL (dB)	-30	-29.4549	-28.9050	-29.6143	-28.2983
Null Depth(dB)	-30	-60.00	-61.04	-61.71	-62.68
No. of Generations		121	255	70	84
CPU time (Sec)		484	1020	280	336

Table 5.60: Computed Array Parameters for Fig. 5.30

controlled elements is increasing from the minimum possible value, which is due to increasing the number of constraints in this optimization. Tables 5.58. and 5.60, show a comparison of array parameters such as directivity and HPBW. The results show that they are almost unchanged. The sidelobe variation (SLV) and SLL in case of partially adaptive arrays are slightly higher than in fully adaptive case, since it is obvious that, the best array parameters are obtained, when all the 30-elements are perturbed, as it affords the greatest control over the array response.

The results of Fig.5.31 (a) and (b) show two nulls, which has been steered to the peaks of the first and fourth sidelobe levels at 6.5° and 16.8° respectively. The results of Fig.5.31 (c) and (d) show three nulls, which has been steered to the peaks of the first, sixth and fourteenth sidelobe levels at 6.5°, 25° and 75° respectively. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.31(a)

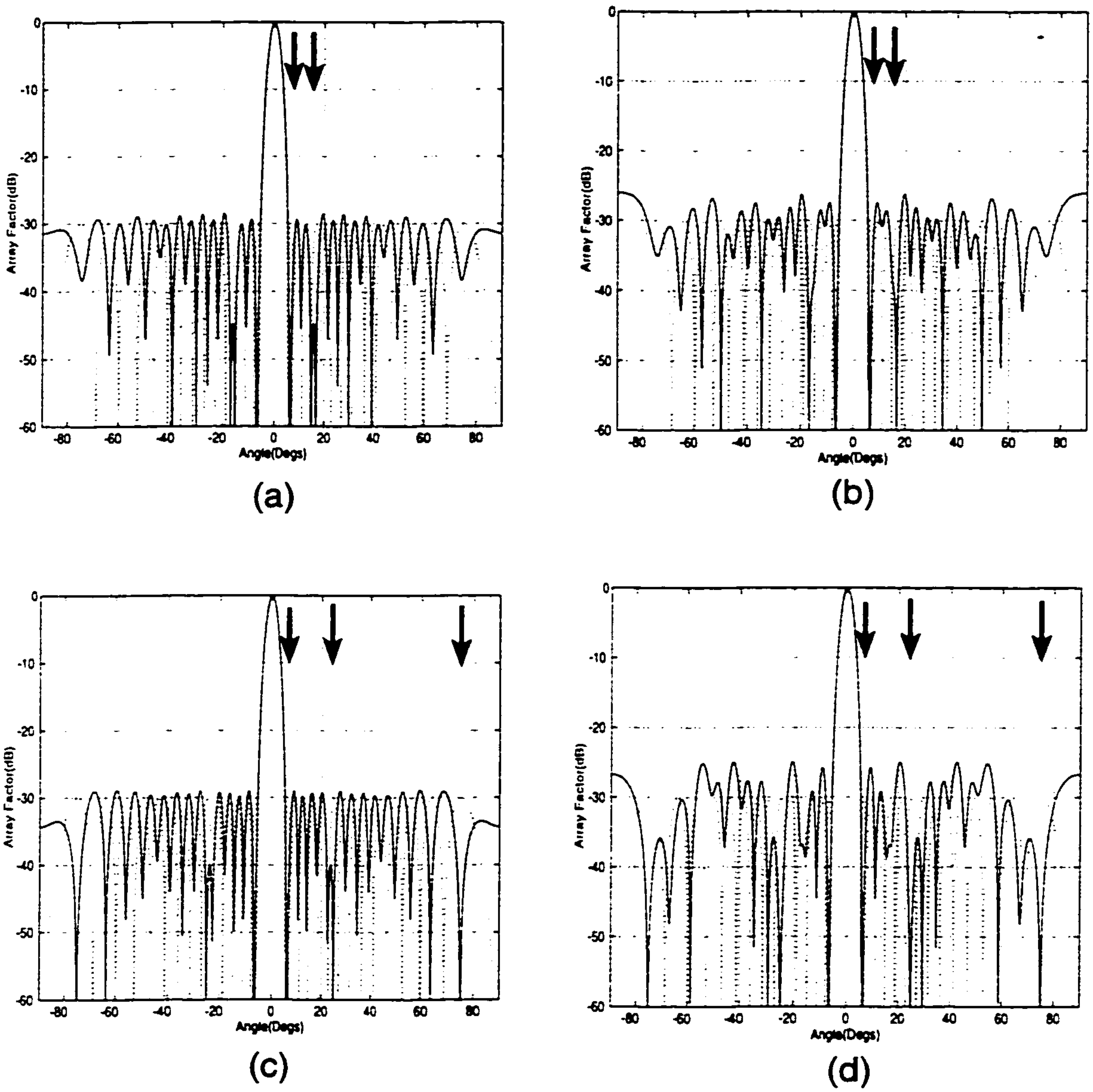


Figure 5.31: Array patterns of 30 element chebyshev array of -30 dB sidelobe level with (a) two nulls imposed at 6.5° and 16.8° respectively, when all elements are controlled and sidelobes restricted to 1.46 dB (b) Minimum (optimum) number of elements are controlled (K=12) with sidelobes restricted to 3.95 dB (c) three nulls imposed at 6.5°, 25° and 75° respectively, when all elements are controlled and sidelobes restricted to 1.08 dB (d) Minimum (optimum) number of elements are controlled (K=15) with sidelobes restricted to 5.05 dB

when all the 30-elements are perturbed and the sidelobe variation is restricted to 1.46 dB. Fig.5.31(b) shows the pattern when the sidelobe variation is restricted to 3.95 dB while achieving the required nulls. K is equal to 12 in this case. Fig.5.31(c) shows the resulting pattern when all the 30-elements are perturbed and the sidelobe variation is restricted to 1.08 dB. Fig.5.31(d) shows the pattern when the sidelobe variation is restricted to 5.05 dB while achieving the required nulls. K is equal to 15 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.31 are given in table5.61 and table5.62 respectively.

The results of Fig.5.32 (a) and (b) show four nulls, which has been steered to the peaks of the second, sixth, twelveth and fourteenth sidelobes at 9.5°,25°,56° and 75° respectively. The results of Fig.5.32 (c) and (d) show five nulls, which has been steered to the peaks of the first, fourth, eighth, twelveth and fourteenth sidelobes at 6.5°, 16.8°, 33.9°, 56° and 75° respectively. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.32(a) when all the 30-elements are perturbed and the sidelobe variation is restricted to 3.95 dB. Fig.5.32(b) shows the pattern when the sidelobe variation is restricted to 6.04 dB while achieving the required nulls. K is equal to 17 in this case. Fig.5.32(c) shows the resulting pattern when all the 30-elements are perturbed and the sidelobe variation is restricted to 2.72 dB. Fig.5.32(d) shows the pattern when the sidelobe variation is restricted to 6.66 dB while achieving the required nulls. K is equal to 18 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.32

ELEMENT Number	Fig. 5.31(a) (Nulls at 6.5°, 16.8°) Full(WSR) Δ_n	Fig. 5.31(b) (Nulls at 6.5°, 16.8°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.31(c) (Nulls at 6.5°, 25°,75°) Full(WSR) Δ_n	Fig. 5.31(d) (Nulls at 6.5°, 25°,75°) Partial(WSR) $\hat{\Delta}_n$
1	-0.2682	-0.1862	-0.3498	-0.1667
2	-0.1155	0	-0.1623	0
3	-0.0387	-0.0761	-0.1374	0
4	-0.0291	0	-0.1508	-0.1337
5	-0.0239	0	-0.0865	-0.1440
6	-0.0078	0	-0.0384	0
7	0.0002	0	0.0001	0
8	0.0751	0.0313	0.0138	0
9	0.0889	0.1592	0.0079	0.0857
10	0.0971	0.2121	0.0256	0.0978
11	0.0968	0.1539	0.0438	0.1138
12	0.0701	0.1290	0.0319	0.1017
13	0.0334	0.0606	0.0274	0
14	0.0099	0.0332	0.0048	0
15	0.0000	0	-0.0004	0
16	0.0000	0	-0.0016	0
17	0.0001	0	-0.0073	0
18	-0.0223	0	-0.0440	-0.0338
19	-0.0373	-0.0341	-0.0640	-0.0979
20	-0.0530	0	-0.0573	-0.1253
21	-0.0834	0	-0.0375	-0.1202
22	-0.0675	0	-0.0287	0
23	-0.0162	0	-0.0232	-0.0092
24	-0.0212	0	-0.0491	0
25	0.0378	0	0.0203	0
26	0.0229	0	0.0917	0
27	0.0008	0	0.1305	0.0689
28	0.0486	0	0.1749	0
29	0.1788	0.2633	0.2136	0.0639
30	0.2991	0.2762	0.2271	0.1372

Table 5.61: Computed element position perturbations for Fig. 5.31 as a function of λ.

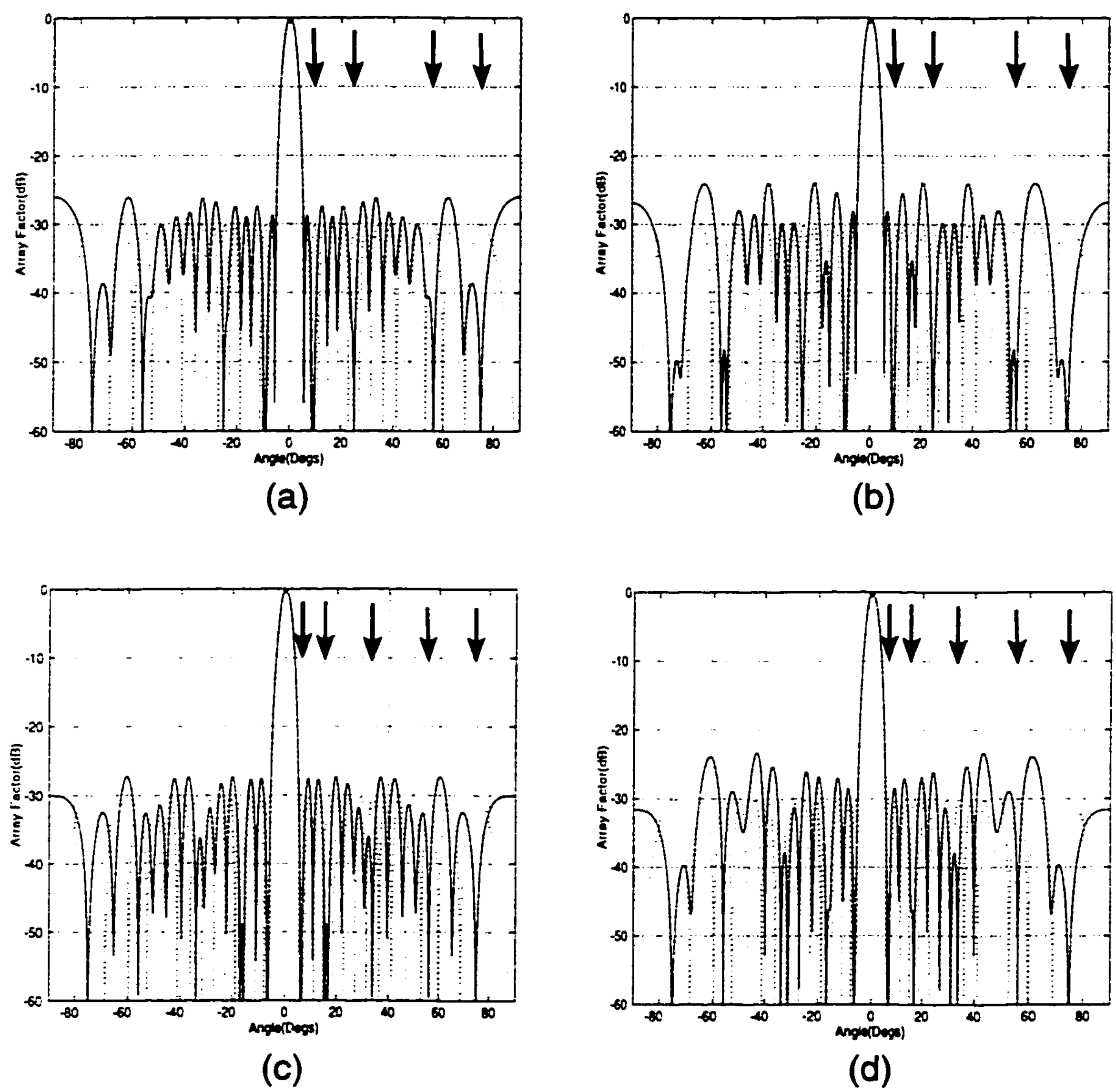


Figure 5.32: Array patterns of 30 element chebyshev array of -30 dB sidelobe level with (a) four nulls imposed at 9.5°,25°,56° and 75°, when all elements are controlled and sidelobes restricted to 3.95 dB (b) Minimum (optimum) number of elements are controlled (K=17) with sidelobes restricted to 6.04 dB (c) five nulls imposed at 6.5°, 16.8°, 33.9°, 56° and 75°, when all elements are controlled and sidelobes restricted to 2.72 dB (d) Minimum (optimum) number of elements are controlled (K=18) with sidelobes restricted to 6.66 dB

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.31(a) (Nulls at 6.5°, 16.8°) Full(WSR)	Fig. 5.31(b) (Nulls at 6.5°, 16.8°) Partial(WSR)	Fig. 5.31(c) (Nulls at 6.5°, 25°,75°) Full(WSR)	Fig. 5.31(d) (Nulls at 6.5°, 25°,75°) Partial(WSR)
Min. Controlled Elements K		30	12	30	15
SLV (dB)		1.4682	3.9514	1.0822	5.0583
DIRECTIVITY	26.2448	26.1391	26.1318	26.4004	26.0477
HPBW (DEG.)	4.1631	4.1584	4.1679	4.1216	4.1678
SLL (dB)	-30	-28.5318	-26.0486	-28.9178	-24.9417
Null Depth(dB)	-30	-60.77	-60.70	-60.00	-60.06
No. of Generations		370	371	1029	306
CPU time (Sec)		1480	1484	4116	1224

Table 5.62: Computed Array Parameters for Fig. 5.31

are given in table 5.63 and table 5.64 respectively.

The results of Fig.5.33 show six nulls, which has been steered to the peaks of the first, fourth, sixth, ninth, twelveth and fourteenth sidelobe levels at 6.5°, 16.8°, 25°, 38.7°, 56° and 75° respectively. The perturbed pattern compared to the initial pattern (dotted) is shown in Fig.5.33(a) when all the 30-elements are perturbed without restricting the sidelobe variation. Fig.5.33(b) shows the perturbed pattern compared to the initial, when all the 30-elements are perturbed and the sidelobe variation is restricted to 4.95 dB. Fig.5.33(c) shows the resulting pattern when the number of controlled elements is reduced to a minimum possible value which is K=19 in this case. The required null has been achieved precisely, but the sidelobe level variation has changed to more than 8 dB. Fig.5.33(d) shows the pattern when the sidelobe variation is restricted to 7.40 dB while achieving the required null. K is equal to 21

ELEMENT Number	Fig. 5.32(a) (Nulls at 9.5°, 25°,56°,75°) Full(WSR) Δ_n	Fig. 5.32(b) (Nulls at 9.5°, 25°,56°,75°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.32(c) (Nulls at 6.5°,16.8°, 33.9°,56°,75°) Full(WSR) Δ_n	Fig. 5.32(d) (Nulls at 6.5°,16.8°, 33.9°,56°,75°) Partial(WSR) $\hat{\Delta}_n$
1	-0.1288	-0.0423	-0.2218	-0.1731
2	-0.0452	-0.0857	-0.1313	-0.0971
3	0.0282	0	-0.0978	-0.0078
4	0.0221	0	-0.0857	0
5	0.0002	0	-0.0629	0
6	0.0905	0.0986	-0.0282	0
7	0.1197	0.0603	-0.0342	0
8	0.0476	0	0.0005	0
9	-0.0017	0	0.0848	0.1334
10	-0.0304	0	0.0840	0.0911
11	-0.0271	-0.0185	0.0777	0.0568
12	-0.0166	0	0.0439	0.0652
13	-0.0540	-0.0716	0.0081	0.0066
14	-0.0570	-0.0690	0.0003	0
15	-0.0263	-0.0752	-0.0037	0
16	0.0409	0.0662	0.0157	0
17	0.0158	0.0796	-0.0008	0
18	0.0497	0.0722	-0.0262	0
19	0.0030	0.0433	-0.0499	-0.0606
20	0.0004	0.0028	-0.0634	-0.0667
21	0.0224	0	-0.0885	-0.0868
22	-0.0057	0	-0.0984	-0.1644
23	-0.0393	0	-0.0060	-0.0116
24	-0.0829	-0.0822	0.0459	0
25	-0.0889	-0.0781	0.0405	0
26	-0.0384	0	0.0696	0.0733
27	-0.0668	0	0.0985	0.0865
28	0.0345	0	0.1143	0.1913
29	0.0668	0.0369	0.1482	0.2142
30	0.0712	0.1044	0.2178	0.2106

Table 5.63: Computed element position perturbations for Fig. 5.32 as a function of λ.

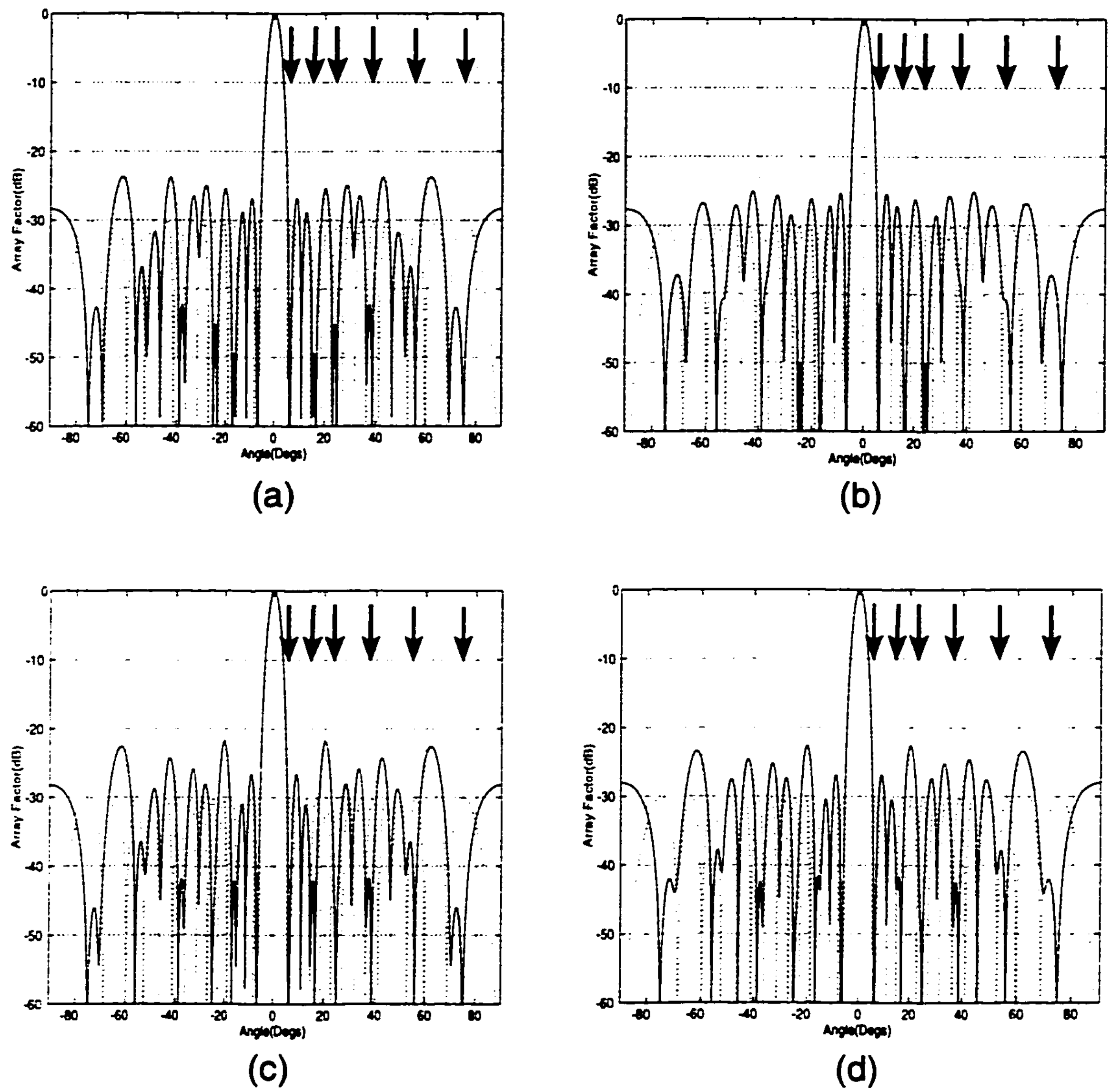


Figure 5.33: Array patterns of 30 element chebyshev array of -30 dB sidelobe level with six nulls imposed at 6.5°, 16.8°, 25°, 38.7°, 56° and 75° (peak of 1^{st}, 4^{th}, 6^{th}, 9^{th}, 12^{th} and 14^{th} sidelobes). (a) all elements are controlled without sidelobe restrictions (b) all elements are controlled with sidelobes restricted to 4.95 dB (c)Minimum (optimum) number of elements are controlled (K=19) without sidelobe restrictions (d)Minimum (optimum) number of elements are controlled (K=21) with sidelobes restricted to 7.40 dB

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.32(a) (Nulls at 9.5°, 25°,56°,75°) Full(WSR)	Fig. 5.32(b) (Nulls at 9.5°, 25°,56°,75°) Partial(WSR)	Fig. 5.32(c) (Nulls at 6.5°,16.8°, 33.9°,56°,75°) Full(WSR)	Fig. 5.32(d) (Nulls at 6.5°,16.8°, 33.9°,56°,75°) Partial(WSR)
Min. Controlled Elements K		30	17	30	18
SLV (dB)		3.9575	6.0439	2.7260	6.6628
DIRECTIVITY	26.2448	26.0646	26.0121	26.2052	25.8474
HPBW (DEG.)	4.1631	4.1703	4.1596	4.1410	4.1607
SLL (dB)	-30	-26.0425	-23.9561	-27.2740	-23.3372
Null Depth(dB)	-30	-60.00	-60.20	-60.00	-60.01
No. of Generations		675	399	1537	546
CPU time (Sec)		2700	1596	6148	2184

Table 5.64: Computed Array Parameters for Fig. 5.32

in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.33 are given in table 5.65 and table 5.66 respectively.

For the case of multiple nulls in 30-element chebyshev partially arrays with sidelobe restriction, the same behavior obtained in case of 8 and 16 element arrays is noticed here. It is observed that as we increase the number of nulls from one to six, the minimum no of controlled elements K, SLL and SLV has also increased. From the tables 5.62, 5.64 and 5.66 comparing the array parameters of partially adaptive array, with the corresponding fully adaptive array, when multiple nulls are imposed, one notices that directivity, and HPBW are almost unchanged. The sidelobe variation (SLV) and SLL in case of partially adaptive arrays is slightly higher than in fully adaptive case, since it is obvious that the best array parameters are obtained, when all the 30-elements are perturbed, as it affords the greatest control over the

ELEMENT Number	Fig. 5.33(a) Full(WOSR) Δ_n	Fig. 5.33(b) Full(WSR) Δ_n	Fig. 5.33(c) Partial(WOSR) $\hat{\Delta}_n$	Fig. 5.33(d) Partial(WSR) $\hat{\Delta}_n$
1	-0.1906	-0.1378	-0.1490	-0.1531
2	-0.1475	-0.0733	-0.1895	-0.0716
3	-0.0820	-0.0668	-0.0887	0
4	-0.0642	-0.1203	0	-0.0266
5	-0.0612	-0.0867	0	-0.0379
6	-0.0301	-0.0667	0	0
7	-0.0068	-0.0489	0	0.0025
8	0.0314	0.0416	0	0
9	0.0739	0.0286	0.0619	0.0267
10	0.0616	0.0463	0.1000	0.0726
11	0.1209	0.1299	0.1686	0.1589
12	0.0826	0.0756	0.1051	0.1006
13	0.0000	0.0001	0	0.0037
14	-0.0004	0.0001	0	0
15	-0.0307	-0.0075	0	-0.0365
16	0.0009	0.0154	0.0404	0
17	0.0005	-0.0002	0	0
18	-0.0009	-0.0090	0	0
19	-0.1269	-0.0770	-0.1050	-0.0938
20	-0.1397	-0.1349	-0.1472	-0.1490
21	-0.0641	-0.0943	-0.0521	-0.0701
22	-0.0696	-0.0563	-0.0052	-0.0391
23	-0.0540	-0.0010	-0.0057	-0.0262
24	-0.0109	0.0294	-0.0048	0
25	0.0399	0.0841	0	0.0088
26	0.0280	0.1493	0.0299	0
27	-0.0008	0.0548	0.1038	0.0612
28	0.0009	0.0671	0.0500	0.1260
29	0.1017	0.0856	0.1135	0.2353
30	0.1165	0.1406	0.1566	0.1627

Table 5.65: Computed element position perturbations for Fig. 5.33 when six nulls are steered at 6.5°,16.8°,25°,38.7°,56° and 75°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.33(a) Full(WOSR)	Fig. 5.33(b) Full(WSR)	Fig. 5.33(c) Partial(WOSR)	Fig. 5.33(d) Partial(WSR)
Min. Controlled Elements K		30	30	19	21
SLV (dB)		6.2713	4.9507	8.1878	7.4051
DIRECTIVITY	26.24	25.8508	26.0469	25.7355	25.7688
HPBW (DEG.)	4.1631	4.1676	4.1467	4.1646	4.1641
SLL (dB)	-30	-23.7287	-25.0493	-21.8122	-22.5949
Null Depth(dB)	-30	-60.03	-60.00	-60.36	-60.00
No. of Generations		1182	462	2544	686
CPU time (Sec)		394	1848	848	2744

Table 5.66: Computed Array Parameters for Fig. 5.33 when six nulls are steered at 6.5°,16.8°,25°,38.7°,56° and 75°.

array response.

5.7.3 Determination of the realizable minimum number of controlled elements

To implement this technique for a single null, the null location which requires the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steering. In the case of 30-element chebyshev partially adaptive arrays a null on the peak of first sidelobe requires the use of 10 controlled elements. Now keeping the location of these 10 controlled elements fixed, a single null is imposed on the peak of sixth sidelobe as shown in Fig.5.34(b) with sidelobes restricted to 5.22 dB . A single null is imposed on the peak of twelveth sidelobe as shown in Fig.5.34(c) with sidelobes

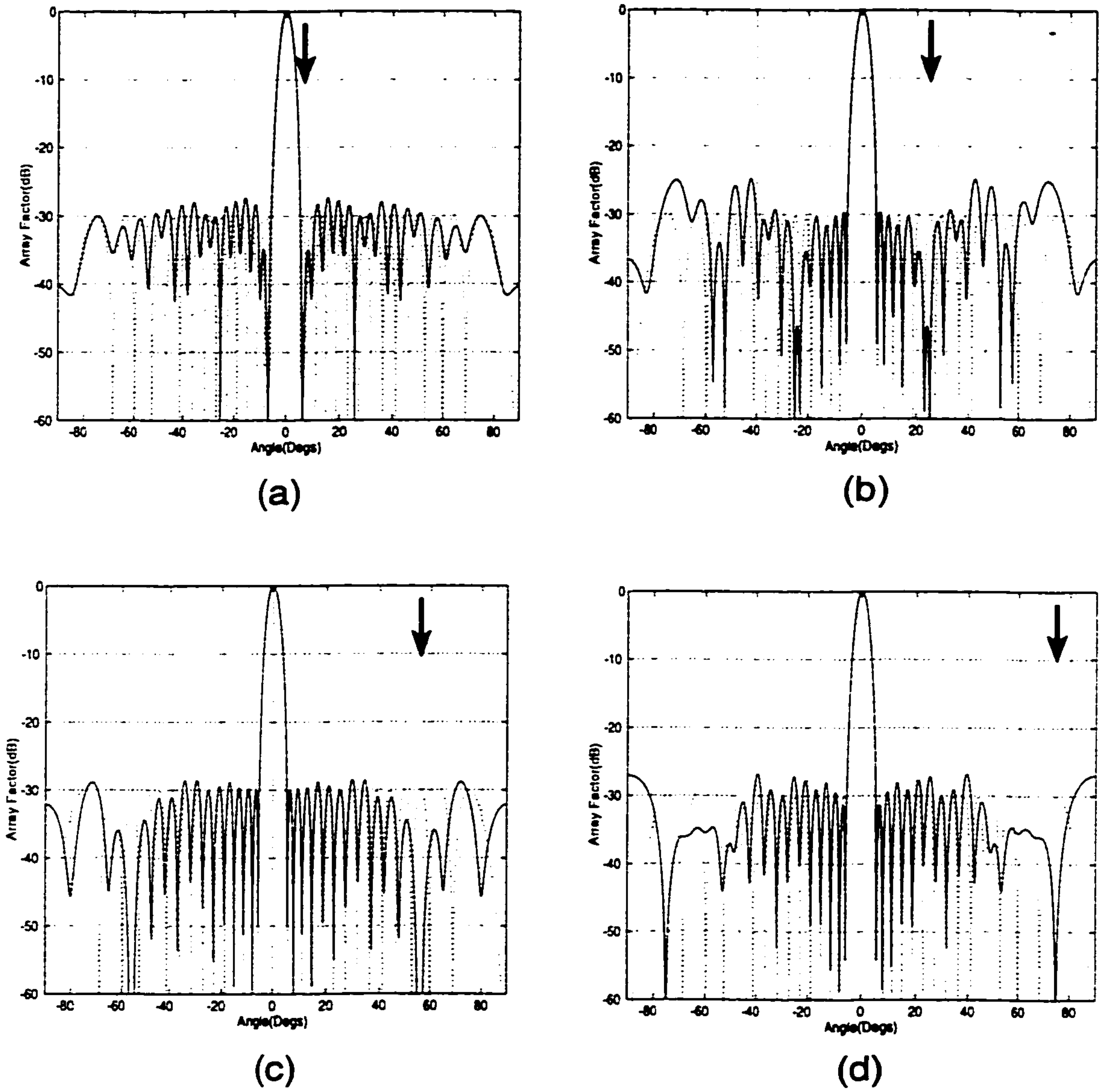


Figure 5.34: Array patterns of 30 element chebyshev array with -30 dB sidelobe level, perturbing 10 fixed elements with (a) one null imposed on the peak of first sidelobe at 6.5° (b) one null imposed on the peak of sixth sidelobe at 25° (c) one null imposed on the peak of twelveth sidelobe at 56° (d) one null imposed on the peak of fourteenth sidelobe at 75°

restricted to 1.46 dB and a single null is imposed on the peak of fourteenth sidelobe as shown in Fig.5.34(c) with sidelobes restricted to 3.22 dB. Therefore perturbing only these 10 fixed elements out of 30-elements, we can steer a single null any where in the sidelobe region of a 30-element chebyshev partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.34 are given in table 5.67 and table 5.68 respectively.

The results given in table 5.68 show that the sidelobe variation (SLV) when the

ELEMENT Number	Fig. 5.34(a) (Null at 6.5°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.34(b) (Null at 25°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.34(c) (Null at 56°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.34(d) (Null at 75°) Partial(WSR) $\hat{\Delta}_n$
1	-0.4898	-0.0905	0.0169	0.0436
2	-0.4714	0.1081	0.0303	0.0028
3	-0.4447	0.0507	-0.0858	-0.1228
4	-0.2962	-0.0484	0.0340	0.0237
10	0.0189	-0.0864	-0.0280	0.0614
26	0.1262	0.0832	0.0464	0.0663
27	0.0877	0.0224	-0.0268	0.0251
28	0.1227	-0.0986	0.0716	0.1467
29	0.3231	-0.0963	-0.0561	-0.0163
30	0.4815	0.1011	-0.0091	0.0045

Table 5.67: Computed element position perturbations for Fig. 5.34 as a function of λ.

null is imposed at sixth, twelveth or fourteenth sidelobe, is higher than the previous results inspite of increasing the no. of controlled elements. This shows that, for a particular null location the SLV depends on the location of the minimum controlled elements.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.34(a) (Null at 6.5°) Partial (WSR)	Fig. 5.34(b) (Null at 25°) Partial (WSR)	Fig. 5.34(c) (Null at 56°) Partial (WSR)	Fig. 5.34(d) (Null at 75°) Partial (WSR)
No. of Controlled Elements		10	10	10	10
SLV (dB)		2.6760	5.2232	1.4629	3.2248
DIRECTIVITY	26.2448	26.7419	26.2957	26.2989	26.2833
HPBW (DEG.)	4.1631	4.0747	4.1550	4.1606	4.1574
SLL (dB)	-30	-27.3240	-24.7768	-28.5371	-26.7752
Null Depth(dB)	-30	-60.53	-60.00	-61.37	-68.06
No. of Generations		550	250	95	206
CPU time (Sec)		2200	1000	380	824

Table 5.68: Computed Array Parameters for Fig. 5.34

To implement this technique for two nulls, the null locations which requires the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steering. In the case of 30-element chebyshev partially adaptive arrays, two nulls on the peaks of first and fourth sidelobe requires the use of 12 controlled elements. Now keeping the location of these 12 controlled elements fixed, two nulls are imposed on the peaks of sixth and ninth sidelobes as shown in Fig.5.35(b) with sidelobes restricted to 2.54 dB. Two nulls are imposed on the peak of eleventh and twelveth sidelobes as shown in Fig.5.35(c) with sidelobes restricted to 2.94 dB and two nulls are imposed on the peak of eleventh and fourteenth sidelobe as shown in Fig.5.35(d) with sidelobes restricted to 3.02 dB. Therefore perturbing only these 12 fixed elements out of 30-elements, we can steer two nulls any where in the sidelobe region

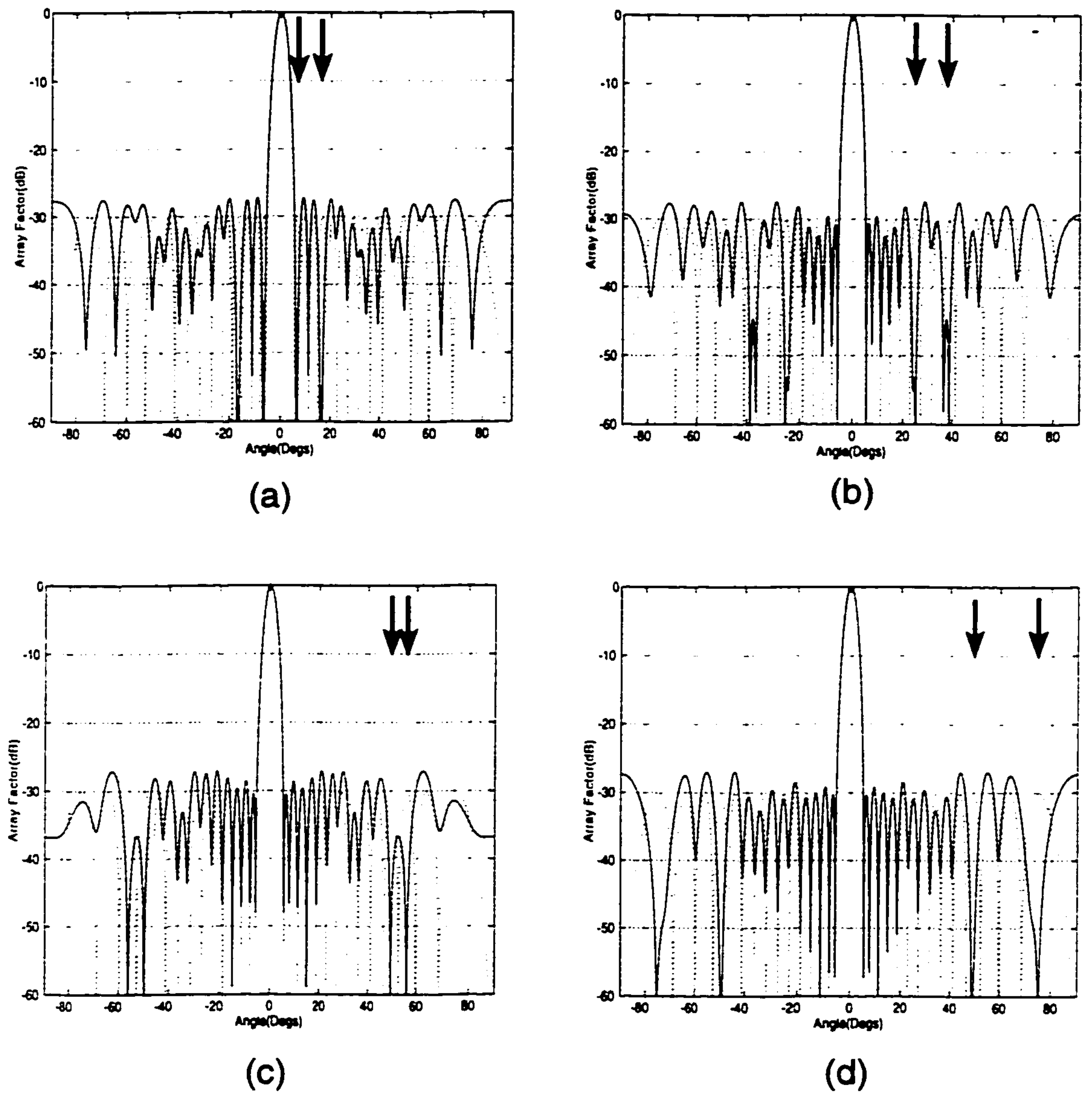


Figure 5.35: Array patterns of 30 element chebyshev array with -30 dB sidelobe level, perturbing 12 fixed elements with (a) two nulls imposed on the peaks of first and fourth sidelobes at 6.5° and 16.8° (b) two nulls imposed on the peak of sixth and ninth sidelobes at 25° and 38.7° (c) two nulls imposed on the peak of eleventh and twelveth sidelobes at 49.6° and 56° (d) two nulls imposed on the peak of eleventh and fourteenth sidelobes at 49.6° and 75°

of a 30-element chebyshev partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.35 are given in table 5.69 and table 5.70 respectively.

ELEMENT Number	Fig. 5.35(a) (Nulls at 6.5°,16.8°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.35(b) (Nulls at 25°,38.7°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.35(c) (Nulls at 49.6°,56°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.35(d) (Nulls at 49.6°,75°) Partial(WSR) $\hat{\Delta}_n$
1	-0.1862	-0.1450	-0.0304	0.0083
3	-0.0761	-0.0074	-0.2451	-0.0283
8	0.0313	0.0137	-0.0585	-0.0225
9	0.1592	-0.0330	0.0250	0.0000
10	0.2121	-0.0391	0.0012	0.0392
11	0.1539	0.0513	-0.0040	-0.0257
12	0.1290	0.0411	0.0000	0.0265
13	0.0606	-0.0012	0.0336	-0.0160
14	0.0332	-0.0293	-0.0046	-0.0015
19	-0.0341	-0.0376	-0.0267	-0.0417
29	0.2633	-0.1482	-0.1108	-0.0706
30	0.2762	0.0229	-0.1277	-0.0016

Table 5.69: Computed element position perturbations for Fig. 5.35 as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.35(a) (Nulls at 6.5°, 16.8°) Partial (WSR)	Fig. 5.35(b) (Nulls at 25°, 38.7°) Partial (WSR)	Fig. 5.35(c) (Nulls at 49.6°, 56°) Partial (WSR)	Fig. 5.35(d) (Nulls at 49.6°, 75°) Partial (WSR)
No. of Controlled Elements		12	12	12	12
SLV (dB)		3.9514	2.5438	2.9499	3.0226
DIRECTIVITY	26.2448	26.1318	26.3010	26.3345	26.2107
HPBW (DEG.)	4.1631	4.1679	4.1612	4.1623	4.1658
SLL (dB)	-30	-26.0486	-27.4562	-27.0501	-26.9774
Null Depth(dB)	-30	-60.70	-60.12	-65.23	-60.00
No. of Generations		371	319	691	169
CPU time (Sec)		1484	1276	2764	676

Table 5.70: Computed Array Parameters for Fig. 5.35

To implement this technique for three nulls, the null locations which requires the highest no. of controlled elements is chosen. Using these controlled elements, nulls are imposed at different locations to test that the system is functional for null steering. In the case of 30-element uniform partially adaptive arrays three nulls on the peaks of first, sixth and fourteenth sidelobes requires the use of 15 controlled elements. Now keeping the location of these 15 controlled elements fixed, three nulls are imposed on the peaks of fifth, eighth and tenth sidelobes as shown in Fig.5.36(b) with sidelobes restricted to 4.07 dB. Three nulls are imposed on the peaks of tenth, eleventh and twelveth sidelobes as shown in Fig.5.36(c) with sidelobes restricted to 3.86 dB and three nulls are imposed on the peaks of second, fourth and sixth sidelobes as shown in Fig.5.36(d) with sidelobes restricted to 5.66 dB. Therefore perturbing only these 15 fixed elements out of 30-elements, we can steer three nulls

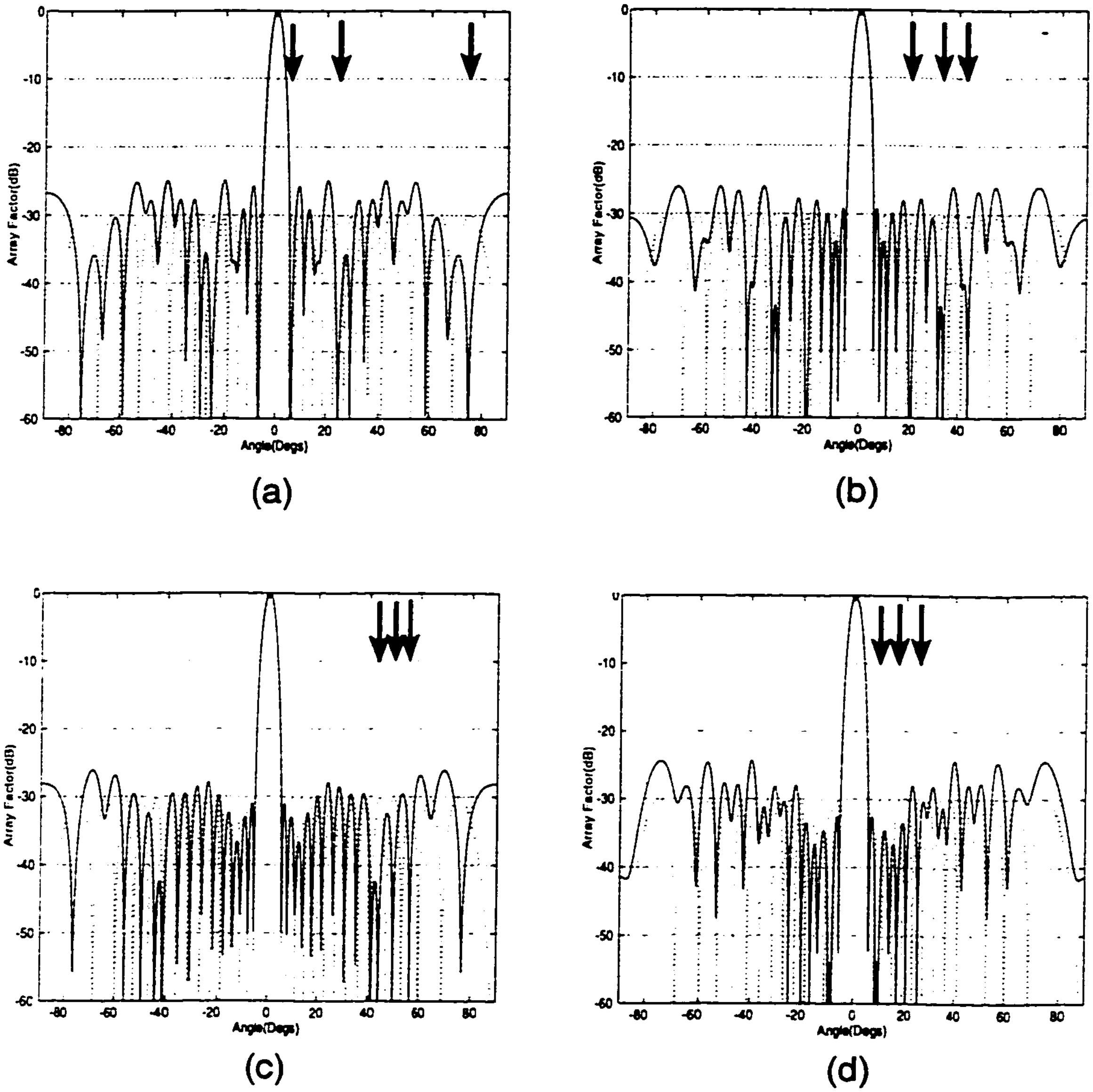


Figure 5.36: Array patterns of 30 element chebyshev array with -30 dB sidelobe level, perturbing 15 fixed elements with (a) three nulls imposed on the peaks of first, sixth and fourteenth sidelobes at 6.5°, 25° and 75° (b) three nulls imposed on the peaks of fifth, eighth and tenth sidelobes at 20.8°, 33.9° and 43.9° (c) three nulls imposed on the peaks of tenth, eleventh and twelveth sidelobes at 43.9°, 49.6° and 56°(d) three nulls imposed on the peaks of second, fourth and sixth sidelobes at 9.5°, 16.8° and 25°

any where in the sidelobe region of an 30-element uniform partially adaptive array. The resulting position perturbations and the array parameters for the patterns of Fig.5.36 are given in table 5.71 and table 5.72 respectively.

The 30-element chebyshev partially adaptive array is suitable for steering upto six

ELEMENT Number	Fig. 5.36(a) (Nulls at 6.5°, 25°,75°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.36(b) (Nulls at 20.8°, 33.9°,43.9°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.36(c) (Nulls at 43.9°, 49.6°,56°) Partial(WSR) $\hat{\Delta}_n$	Fig. 5.36(d) (Nulls at 9.5°, 16.8°,25°) Partial(WSR) $\hat{\Delta}_n$
1	-0.1667	-0.0955	-0.1008	-0.4911
4	-0.1337	-0.0437	0.0830	-0.0068
5	-0.1440	-0.0124	0.0457	0.0701
9	0.0857	0.0002	-0.0126	0.0556
10	0.0978	-0.0125	0.0118	-0.0043
11	0.1138	-0.0692	0.0000	-0.0152
12	0.1017	-0.0546	-0.0035	-0.0259
18	-0.0338	-0.0380	-0.0265	0.0568
19	-0.0979	0.0550	0.0000	0.0034
20	-0.1253	0.0492	-0.0040	0.0093
21	-0.1202	-0.0122	-0.0000	0.0307
23	-0.0092	-0.0530	0.0151	-0.0486
27	0.0689	-0.0182	-0.1113	-0.0548
29	0.0639	-0.0727	0.0847	0.1021
30	0.1372	0.1042	0.2822	0.4857

Table 5.71: Computed element position perturbations for Fig. 5.36 as a function of λ.

nulls, because steering seven nulls requires more than 21 controlled elements out of 30.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.36(a) (Nulls at 6.5°, 25°,75°) Partial (WSR)	Fig. 5.36(b) (Nulls at 20.8°, 33.9°,43.9°) Partial (WSR)	Fig. 5.36(c) (Nulls at 43.9°, 49.6°,56°) Partial (WSR)	Fig. 5.36(d) (Nulls at 9.5°, 16.8°,25°) Partial (WSR)
No. of Controlled Elements		15	15	15	15
SLV (dB)		5.0583	4.0770	3.8632	5.6694
DIRECTIVITY	26.2448	26.0477	26.2580	26.3498	26.3664
HPBW (DEG.)	4.1631	4.1678	4.1506	4.1546	4.1273
SLL (dB)	-30	-24.9417	-25.9230	-26.1368	-24.3306
Null Depth(dB)	-30	-60.06	-60.00	-65.28	-60.10
No. of Generations		306	2804	1276	600
CPU time (Sec)		1224	11216	5104	2400

Table 5.72: Computed Array Parameters for Fig. 5.36

5.8 Summary

The 8-element partially adaptive array is suitable to steer only a single null. For the uniform array, the minimum no. of controlled elements vary between 2 and 4 depending on the location of the steered null. In order to be able to steer a single null over the whole sidelobe region, then 4 controlled elements out of 8 should be used. In order to be able to steer two nulls over most of the sidelobe region, then 6 controlled elements out of 8 should be used. For the chebyshev array, the minimum no. of controlled elements vary between 2 and 3 depending on the location of the steered null. In order to be able to steer a single null over the whole sidelobe region, then 3 controlled elements out of 8 should be used. In order to be able to steer two nulls over the whole sidelobe region, then 5 controlled elements out of 8 should be used. Summary of the results for 8 element uniform and chebyshev partially adaptive array is given in table 5.73.

The 16-element uniform partially adaptive array is suitable to steer up to two nulls. The minimum number of controlled elements, to steer a single null in the sidelobe region varies between 3 and 7 depending on the location of the steered null. In order to be able to steer a single null over most of the sidelobe region, then 7 controlled elements out of 16 should be used. For two nulls, the minimum no. of controlled elements vary between 7 and 8 depending on the locations of the steered nulls. In order to be able to steer two nulls over most of the sidelobe region, then 8 controlled

	No. of Nulls	Required range of controlled elements depending upon the location of imposed nulls	Required no. of controlled elements so that nulls can be steered any where in the sidelobe region	
			No. of controlled elements	Location of elements
8 Element Uniform array	1	2-4	4	1,3,6,8
	2	6	6	1,3,4,6,7,8
8 Element Chebyshev array	1	2-3	3	1,7,8
	2	5	5	1,3,6,7,8

Table 5.73: Summary of the results for 8 element uniform and chebyshev partially adaptive arrays

elements out of 16 should be used. In order to be able to steer three and four nulls over most of the sidelobe region, then 11 and 12 controlled elements out of 16 should be used respectively.

The 16-element chebyshev partially adaptive array is suitable to steer up to four nulls. The minimum number of controlled elements, to steer a single null in the sidelobe region varies between 3 and 6 depending on the location of the steered null. In order to be able to steer a single null over the whole sidelobe region, then 6 controlled elements out of 16 should be used. For two nulls, the minimum no. of controlled elements vary between 6 and 8 depending on the locations of the steered nulls. In order to be able to steer two or three nulls over the whole sidelobe region, then 8 controlled elements out of 16 should be used. In order to be able to steer four nulls over the whole sidelobe region, then 9 controlled elements out of 16 should be used. Summary of the results for 16 element uniform and chebyshev partially adaptive array is given in table 5.74.

The 30-element uniform partially adaptive array is suitable to steer up to three

	No. of Nulls	Required range of controlled elements depending upon the location of imposed nulls	Required no. of controlled elements so that nulls can be steered any where in the sidelobe region: No. of controlled elements	Required no. of controlled elements so that nulls can be steered any where in the sidelobe region: Location of elements
16 Element Uniform array	1	3-7	7	1,3,4,6,7,8,16
	2	7-8	8	1,9,10,11,13,14,15,16
	3	11	11	1,3,4,6,7,8,10,11,13,14,16
	4	12	12	1,3,4,6,7,8,9,10,11,13,14,16
16 Element Chebyshev array	1	3-6	6	3,4,7,13,14,16
	2	6-8	8	1,3,7,8,12,13,15,16
	3	8	8	1,3,7,8,9,10,14,16
	4	9	9	1,3,5,6,9,11,13,14,16

Table 5.74: Summary of the results for 16 element uniform and chebyshev partially adaptive arrays

nulls. The minimum number of controlled elements, to steer a single null in the sidelobe region varies between 6 and 16 depending on the location of the steered null. In order to be able to steer a single null over most of the sidelobe region, then 16 controlled elements out of 30 should be used. For two nulls, the minimum no. of controlled elements vary between 10 and 18 depending on the locations of the steered nulls. In order to be able to steer two nulls over most of the sidelobe region, then 18 controlled elements out of 30 should be used. In order to be able to steer three, four, five and six nulls over most of the sidelobe region, then 21, 23, 24 and 26 controlled elements out of 30 should be used respectively.

The 30-element chebyshev partially adaptive array is suitable to steer up to six nulls. The minimum number of controlled elements, to steer a single null in the sidelobe region varies between 6 and 10 depending on the location of the steered null. In order to be able to steer a single null over the whole sidelobe region, then 6 controlled elements out of 30 should be used. For two nulls, the minimum no. of controlled elements vary between 10 and 12 depending on the locations of the steered nulls. In order to be able to steer two nulls over the whole sidelobe region, then 12 controlled elements out of 30 should be used. In order to be able to steer three, four, five and six nulls over the whole sidelobe region, then 15, 17, 18 and 21 controlled elements out of 30 should be used respectively. Summary of the results for 30 element uniform and chebyshev partially adaptive array is given in table 5.75.

Figure 5.37 shows the percentage reduction in the no. of controlled elements Vs. No. of array elements of uniform array for the cases of one, two, three and four nulls. It is observed that the percentage reduction in the no. of controlled elements decreases as the no. of array element decreases. But as the array size increases, more degrees of freedom are available and hence, percentage reduction in the no. of controlled Elements increases. Also as the number of nulls increases the percentage reduction in the no. of controlled Elements decreases.

Figure 5.38 shows the percentage reduction in the no. of controlled elements Vs. No. of array elements of chebyshev array for the cases of one, two, three and four nulls. The same behavior obtained in the case of uniform array is observed here. It is observed that the percentage reduction in the no. of controlled elements decreases

	No. of Nulls	Required range of controlled elements depending upon the location of imposed nulls	Required no. of controlled elements so that nulls can be steered any where in the sidelobe region: No. of controlled elements	Location of elements
30 Element Uniform array	1	6-16	16	1,2,11,12,13,17,18,19,20 21,23,24,25,26,29,30
	2	10-18	18	1,2,4,5,6,7,12,13,14,15,17 18,19,24,25,26,29,30
	3	21	21	1,2,3,4,5,6,7,8,11,12,13,14 17,18,19,24,25,26,27,28,30
	4	23	23	1,2,3,4,5,6,7,8,9 11,12,13,14,15,17,18,19 20,23,24,25,26,30
	5	24	24	1,2,4,5,6,7,8,9,11 12,13,14,16,17,18,19,20 23,24,25,26,27,29,30
	6	26	26	1,2,4,5,6,7,8,9,11,12 13,14,15,16,17,18,19,20 23,24,25,26,27,28,29,30
30 Element Chebyshev array	1	6-10	10	1,2,3,4,10,26,27,28,29,30
	2	10-12	12	1,3,8,9,10,11,12,13,14 19,29,30
	3	15	15	1,4,5,9,10,11,12,18,19 20,21,23,27,29,30
	4	17	17	1,2,6,7,11,13,14,15,16 17,18,19,20,24,25,29,30
	5	18	18	1,2,3,9,10,11,12,13,19 20,21,22,23,26,27,28,29,30
	6	21	21	1,2,4,5,7,9,10,11,12, 13,15,19,20,21,22 23,25,27,28,29,30

Table 5.75: Summary of the results for 30 element uniform and chebyshev partially adaptive arrays

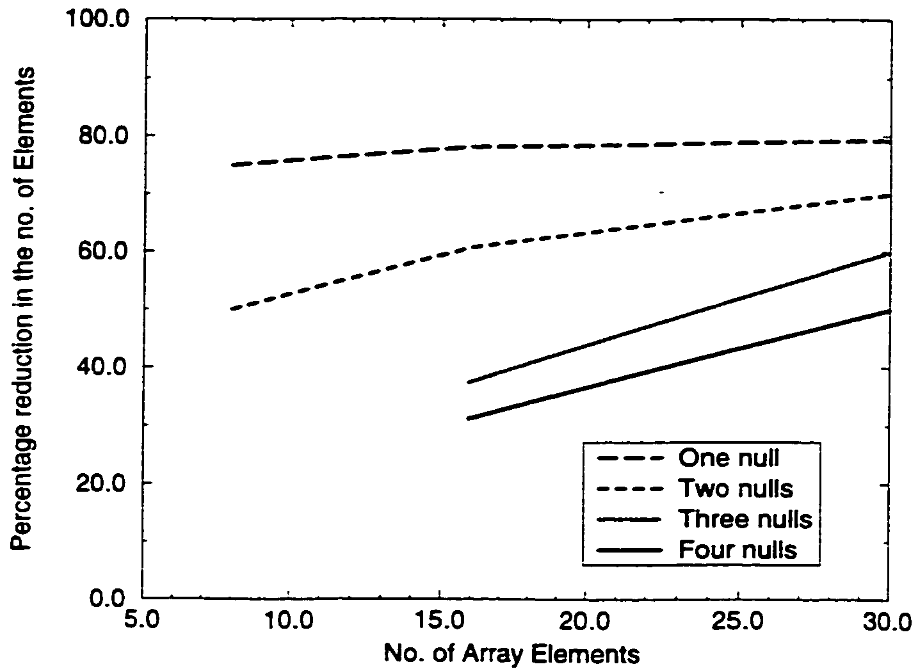


Figure 5.37: Percentage reduction in the no. of controlled elements Vs. No. of array elements of uniform array for the cases of one, two, three and four nulls.

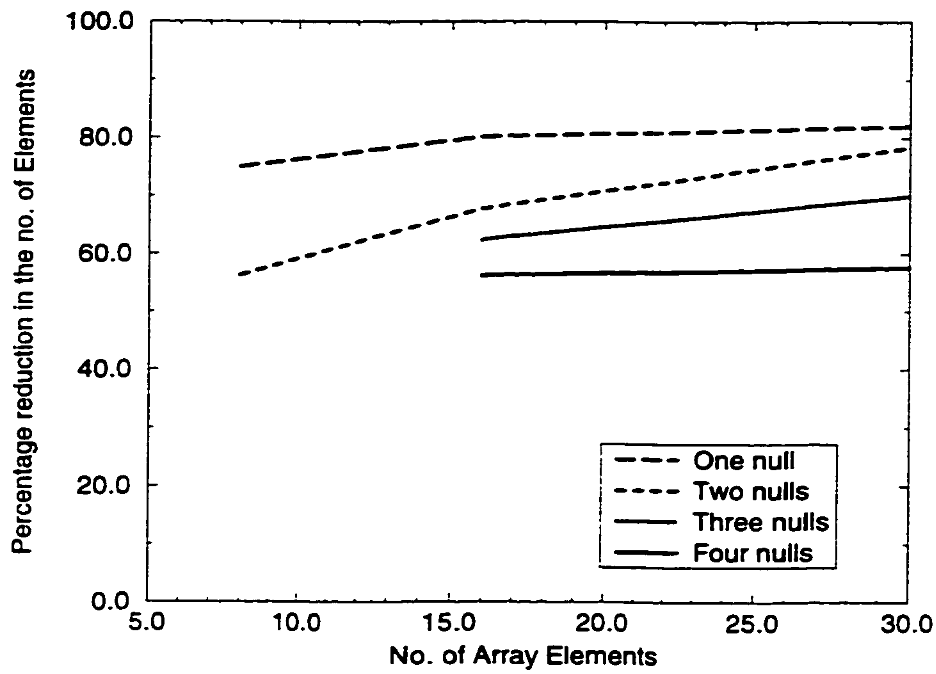


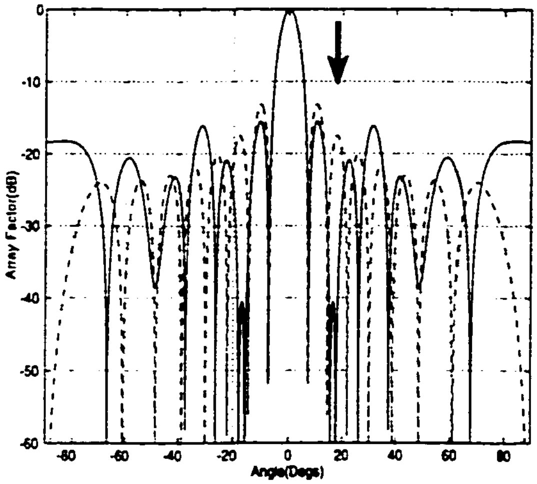
Figure 5.38: Percentage reduction in the no. of controlled elements Vs. The no. of array elements of chebyshev array for the cases of one, two, three and four nulls.

as the no. of array element decreases. But as the array size increases, more degrees of freedom are available and hence, percentage reduction in the no. of controlled elements increases. Also as the number of nulls increases the percentage reduction in the no. of controlled elements decreases. In the case of chebyshev array the percentage reduction in the no. of controlled elements is more compared to the case of uniform array, this is because of the low sidelobe level of chebyshev array. But the price we pay here for choosing array with low sidelobe level is a wider half-power beamwidth and low directivity.

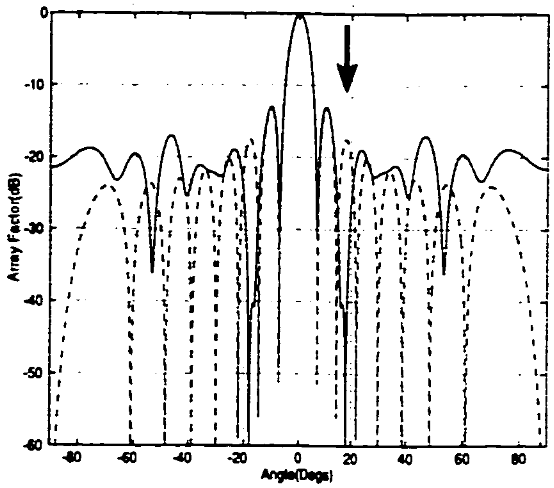
5.9 Comparison

In this section, the results obtained from the proposed GA technique are compared with the typical examples obtained by analytical method from Al-Mushcab R.T. thesis [25]. The results of Fig.5.39 show comparison of the array patterns of 16 element uniform array with one null, which has been steered to the peak of the second sidelobe level at 17°. The perturbed pattern compared to the initial pattern (dashed) is shown in Fig.5.39(a) where the perturbed pattern is obtained analytically, from R.T. thesis by controlling selected 8 edge elements. By this analytical method we are able to achieve a null depth of only -51 dB in the given direction as compared to the required null depth of -60 dB. Fig.5.39(b) shows the resulting pattern obtained by GA, when the number of controlled elements is reduced to a minimum possible value of K=4. The required null has been achieved precisely in the given direction with the required null depth of -60 dB. Fig.5.39(c) shows the pattern obtained by GA, when the sidelobe variation is restricted to 2.52 dB, while achieving the required null. K is equal to 6 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.39 are given in table 5.76 and table 5.77 respectively.

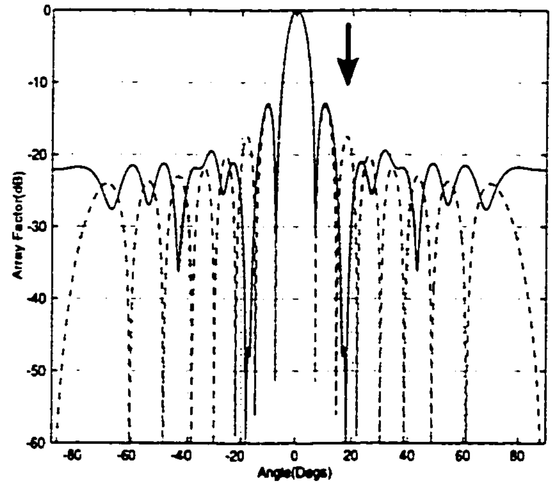
The results of Fig.5.40 show comparison of the array patterns of 16 element chebyshev array with -20 dB sidelobe level and one null, which has been steered to the peak of the second sidelobe level at 17°. The perturbed pattern compared to



(a)



(b)



(c)

Figure 5.39: Comparison of the array patterns of 16 element uniform array with one null imposed at 17° (a) Resultant pattern obtained analytically from R.T. thesis by controlling 8 edge elements (b) Pattern obtained by GA by controlling minimum (optimum) number of elements (K=4) without sidelobe restrictions (c) Pattern obtained by GA by controlling minimum (optimum) number of elements (K=6) with sidelobes restricted to 2.52 dB

ELEMENT Number	Fig. 5.39(a) RT thesis $\hat{\Delta}_n$	Fig. 5.39(b) GA (WOSR) $\hat{\Delta}_n$	Fig. 5.39(c) GA (WSR) $\hat{\Delta}_n$
1	-0.2054	-0.2810	-0.2676
2	-0.0066	0	0
3	0.1980	0	0.1708
4	0.2302	0	0
5	0	0	0
6	0	0	0
7	0	0	0
8	0	0	0
9	0	0.2402	0.2229
10	0	0.4953	0.3060
11	0	0	0.1956
12	0	0	0
13	-0.2302	0	0
14	-0.1980	0	0
15	0.0066	0	0
16	0.2054	0.2508	0.1731

Table 5.76: Comparison of computed element position perturbations for Fig. 5.39 when a single null is steered at 17°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.39(a) RT thesis	Fig. 5.39(b) GA (WOSR)	Fig. 5.39(c) GA (WSR)
Min. Controlled Elements K		8	4	6
SLV (dB)		5.8070	6.2438	2.5289
DIRECTIVITY	16	15.8582	16.7291	16.9205
HPBW (DEG.)	6.3588	6.4164	6.1752	6.2276
SLL (dB)	-13.22	-15.5517	-12.9540	-12.8346
Null Depth(dB)	-17.49	-51.19	-60.00	-60.03
No. of Generations			120	294
CPU time (Sec)			40	1176

Table 5.77: Comparison of computed Array Parameters for Fig. 5.39 when a single null is steered at 17°.

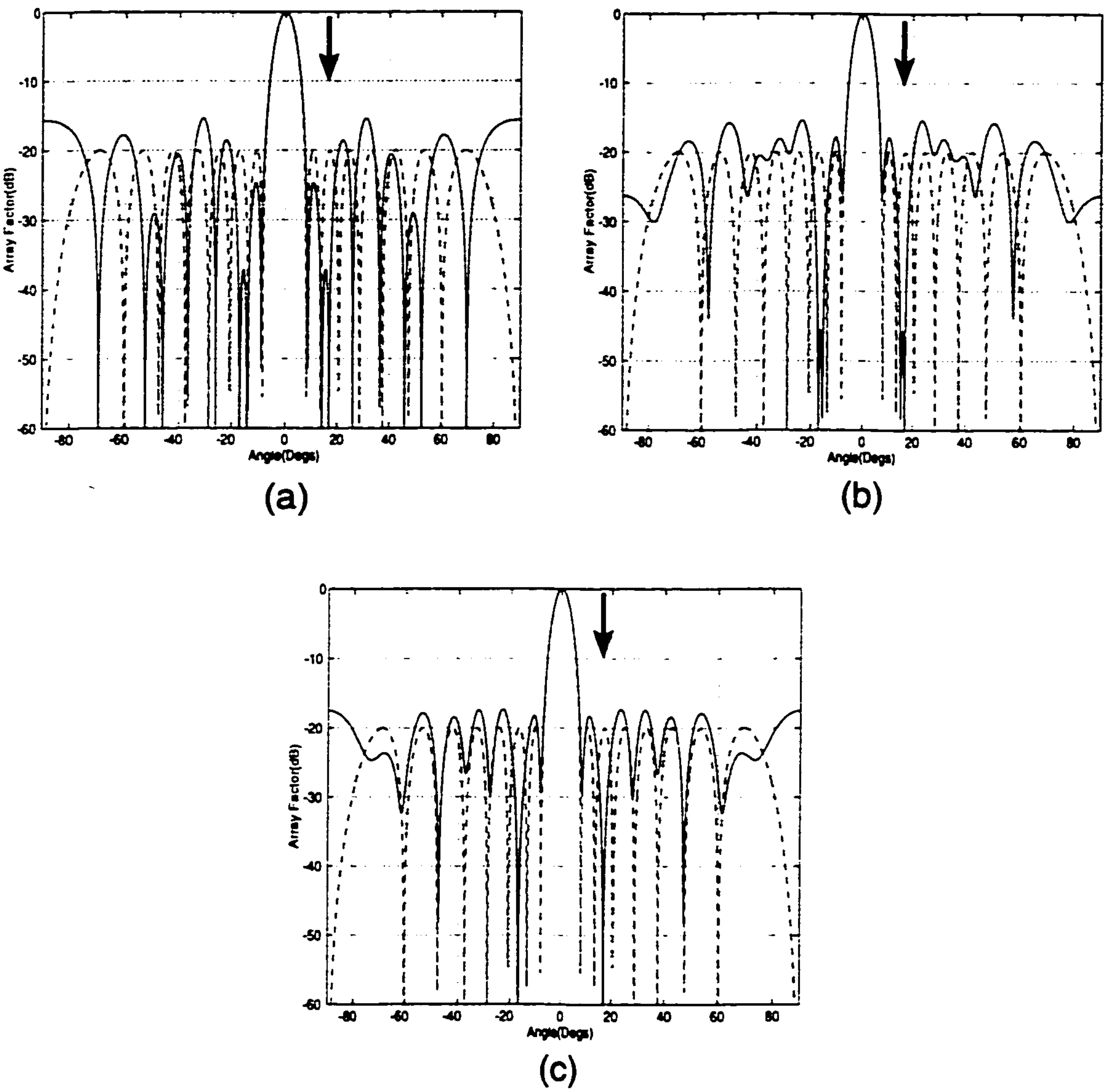


Figure 5.40: Comparison of the array patterns of 16 element chebyshev array with -20 dB sidelobe level and one null imposed at 17° (a) Resultant pattern obtained analytically from R.T. thesis by controlling 8 edge elements (b) Pattern obtained by GA by controlling minimum (optimum) number of elements (K=3) without sidelobe restrictions (c) Pattern obtained by GA by controlling minimum (optimum) number of elements (K=5) with sidelobes restricted to 2.67 dB

the initial pattern (dashed) is shown in Fig.5.40(a) where the perturbed pattern is obtained analytically, from R.T. thesis by controlling selected 8 edge elements. By this analytical method we are able to achieve a null depth of -58 dB in the given direction as compared to the required null depth of -60 dB. Fig.5.40(b) shows the resulting pattern obtained by GA, when the number of controlled elements is reduced to a minimum possible value of K=3. The required null has been achieved precisely in the given direction with the required null depth of -60 dB. Fig.5.40(c) shows the pattern obtained by GA, when the sidelobe variation is restricted to 2.67 dB while achieving the required null. K is equal to 5 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.40 are given in table 5.78 and table 5.79 respectively.

The results of Fig.5.41 show comparison of the array patterns of 20 element uniform array with two nulls, which has been steered to the peak of the seventh and ninth sidelobe level at 49° and 72° respectively. The perturbed pattern compared to the initial pattern (dashed) is shown in Fig.5.41(a) where the perturbed pattern is obtained analytically, from R.T. thesis by controlling selected 16 center elements. Fig.5.41(b) shows the resulting pattern obtained by GA, when the number of controlled elements is reduced to a minimum possible value of K=6. The required nulls has been achieved precisely in the given directions with the required null depths of -60 dB. Fig.5.41(c) shows the pattern obtained by GA, when the sidelobe variation is restricted to 1.8 dB, while achieving the required null. K is equal to 9 in this case.

ELEMENT Number	Fig. 5.40(a) RT thesis $\hat{\Delta}_n$	Fig. 5.40(b) GA (WOSR) $\hat{\Delta}_n$	Fig. 5.40(c) GA (WSR) $\hat{\Delta}_n$
1	-0.1751	0	0
2	0.0551	0	0.4408
3	0.2080	0	0.4276
4	0.2178	0	0.2708
5	0	0	0
6	0	0	0
7	0	-0.3858	-0.0910
8	0	-0.2705	0
9	0	0	0
10	0	0	0.0769
11	0	0	0
12	0	0	0
13	-0.2178	0	0
14	-0.2080	0	0
15	-0.0551	0	0
16	0.1751	0.2616	0

Table 5.78: Comparison of computed element position perturbations for Fig. 5.40 when a single null is steered at 17°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.40(a) RT thesis	Fig. 5.40(b) GA (WOSR)	Fig. 5.40(c) GA (WSR)
Min. Controlled Elements K		8	3	5
SLV (dB)		4.6216	4.6256	2.6755
DIRECTIVITY	15.3820	14.8210	16.5659	15.1455
HPBW (DEG.)	6.7772	6.8521	6.6316	6.9747
SLL (dB)	-20	-15.3784	-15.3744	-17.3245
Null Depth(dB)	-20	-58.59	-60.00	-60.04
No. of Generations			73	110
CPU time (Sec)			24.3	440

Table 5.79: Comparison of computed Array Parameters for Fig. 5.40 when a single null is steered at 17°.

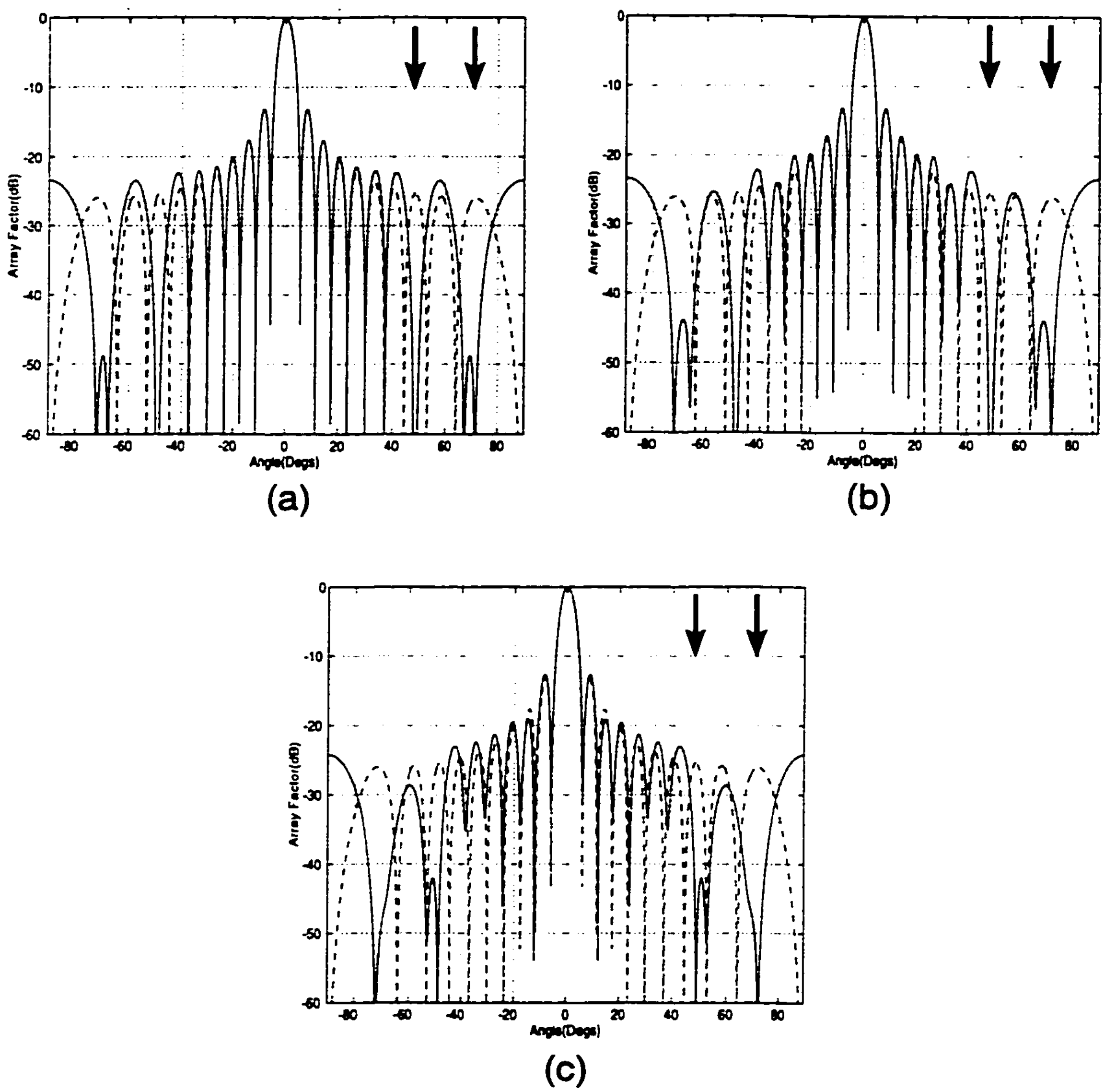


Figure 5.41: Comparison of the array patterns of 20 element uniform array with two nulls imposed at 49° and 72° (a) Resultant pattern obtained analytically from R.T. thesis by controlling 16 center elements (b) Pattern obtained by GA by controlling minimum (optimum) number of elements (K=6) without sidelobe restrictions (c) Pattern obtained by GA by controlling minimum (optimum) number of elements (K=9) with sidelobes restricted to 1.8 dB

The resulting position perturbations and the array parameters for the patterns of Fig.5.41 are given in table 5.80 and table 5.81 respectively.

The results of Fig.5.42 show comparison of the array patterns of 20 element

ELEMENT Number	Fig. 5.41(a) RT thesis $\hat{\Delta}_n$	Fig. 5.41(b) GA (WOSR) $\hat{\Delta}_n$	Fig. 5.41(c) GA (WSR) $\hat{\Delta}_n$
1	0	0	0
2	0	0	0
3	-0.0339	-0.0611	-0.0724
4	0.0188	0	0
5	0.0015	0	0
6	-0.0143	0	-0.0075
7	0.0114	0.0230	0
8	0.0066	0	0
9	-0.0302	-0.0067	0
10	0.0464	0.0674	0
11	-0.0464	-0.0549	0
12	0.0302	0	0.1211
13	-0.0066	0	0
14	-0.0114	0	0
15	0.0143	0	-0.1154
16	-0.0015	0	-0.1115
17	-0.0188	0	-0.2492
18	0.0339	0.0552	-0.1886
19	0	0	-0.2465
20	0	0	-0.2255

Table 5.80: Comparison of computed element position perturbations for Fig. 5.41 when two nulls are steered at 49° and 72°. Perturbations are given as a function of λ.

chebyshev array with -30 dB sidelobe level and two nulls, which has been steered to the peak of the seventh and ninth sidelobe level at 49° and 72° respectively. The perturbed pattern compared to the initial pattern (dashed) is shown in Fig.5.42(a)

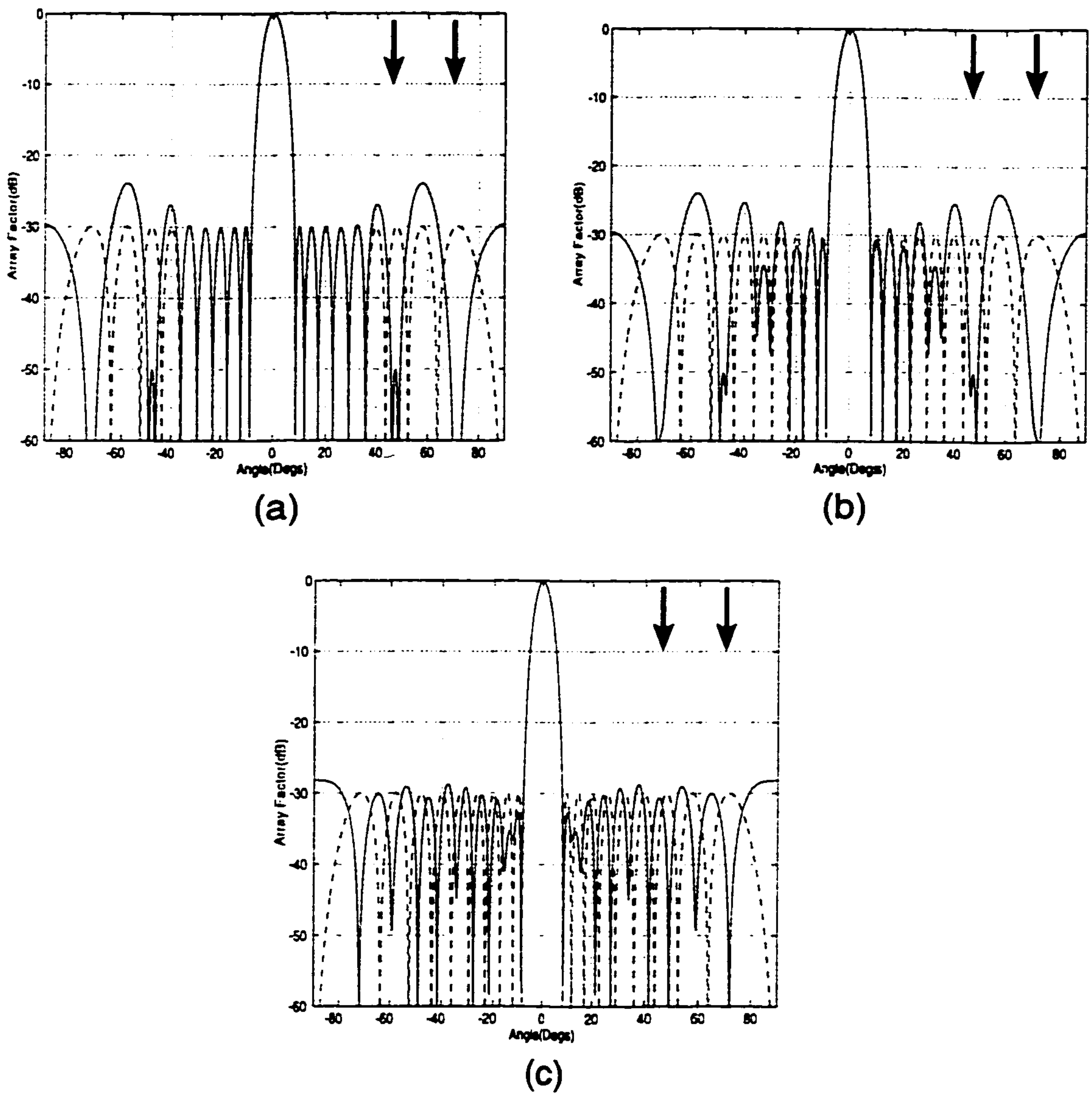


Figure 5.42: Comparison of the array patterns of 20 element chebyshev array with -30 dB sidelobe level and two nulls imposed at 49° and 72° (a) Resultant pattern obtained analytically from R.T. thesis by controlling 14 center elements (b) Pattern obtained by GA by controlling minimum (optimum) number of elements (K=3) without sidelobe restrictions (c) Pattern obtained by GA by controlling minimum (optimum) number of elements (K=4) with sidelobes restricted to 1.8 dB

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.41(a) RT thesis	Fig. 5.41(b) GA (WOSR)	Fig. 5.41(c) GA (WSR)
Min. Controlled Elements K		16	6	9
SLV (dB)		2.5930	2.6536	1.8008
DIRECTIVITY	20	19.8817	19.9014	19.7956
HPBW (DEG.)	5.0831	5.0783	5.0718	5.1985
SLL (dB)		-13.2176	-13.3162	-12.6577
Null Depth(dB)		-62.26	-61.40	-61.28
No. of Generations			562	389
CPU time (Sec)			187.3	1556

Table 5.81: Comparison of computed Array Parameters for Fig. 5.41 when two nulls are steered at 49° and 72°.

where the perturbed pattern is obtained analytically, from R.T. thesis by controlling selected 14 center elements. Fig.5.42(b) shows the resulting pattern obtained by GA, when the number of controlled elements is reduced to a minimum possible value of K=3. The required nulls has been achieved precisely in the given directions with the required null depths of -60 dB. Fig.5.42(c) shows the pattern obtained by GA, when the sidelobe variation is restricted to 1.8 dB, while achieving the required null. K is equal to 4 in this case. The resulting position perturbations and the array parameters for the patterns of Fig.5.42 are given in table 5.82 and table 5.83 respectively.

ELEMENT Number	Fig. 5.42(a) RT thesis $\hat{\Delta}_n$	Fig. 5.42(b) GA (WOSR) $\hat{\Delta}_n$	Fig. 5.42(c) GA (WSR) $\hat{\Delta}_n$
1	0	0	-0.3925
2	0	0	-0.1123
3	0	0	0.0014
4	0.0052	0	0
5	0.0013	0	0
6	-0.0073	0	0
7	0.0070	0	0
8	0.0021	0	-0.0114
9	-0.0159	0	0
10	0.0261	0.0337	0
11	-0.0261	-0.0469	0
12	0.0159	0	0
13	-0.0021	0	0
14	-0.0070	-0.0136	0
15	0.0073	0	0
16	-0.0013	0	0
17	-0.0052	0	0
18	0	0	0
19	0	0	0
20	0	0	0

Table 5.82: Comparison of computed element position perturbations for Fig. 5.42 when two nulls are steered at 49° and 72°. Perturbations are given as a function of λ.

ARRAY PARAMETERS	INITIAL VALUES	Fig. 5.42(a) RT thesis	Fig. 5.42(b) GA (WOSR)	Fig. 5.42(c) GA (WSR)
Min. Controlled Elements K		14	3	4
SLV (dB)		6.0453	5.9789	1.8054
DIRECTIVITY	17.3497	17.2756	17.2467	17.6301
HPBW (DEG.)	6.3278	6.3287	6.3319	6.2752
SLL (dB)	-30	-23.9547	-24.0211	-28.1946
Null Depth(dB)	-30	-76.65	-60.08	-60.10
No. of Generations			192	223
CPU time (Sec)			64	892

Table 5.83: Comparison of computed Array Parameters for Fig. 5.42 when two nulls are steered at 49° and 72°.

5.9.1 Conclusion

The analytical method of designing partially adaptive array in R.T. thesis [25] starts by fixing the location and number of perturbed elements and it depends on trial & error method. Therefore this method does not give the minimum number of elements K that can be controlled to steer the nulls in the required interference directions. The proposed GA technique starts by selecting the location of perturbed element randomly and the number of perturbed elements are increased one at a time until the performance specifications are attained. Therefore the proposed GA technique gives the minimum (optimum) number of elements K along with their locations, that can be controlled to steer the nulls in the required interference directions. From the above comparisons it is clear that the proposed GA technique gives less number of controlled elements K compared to the analytical method in R.T. thesis.

The analytical method does not give the required null depth of -60 dB in the given directions, particularly when the null is imposed close to the main beam. Where as the proposed GA technique give the nulls precisely in the given directions with the required null depths of -60 dB irrespective of the location of nulls. It is also observed that using the proposed GA technique the change in array parameters such as HPBW and directivity is negligible.

Chapter 6

Conclusions and future work

This chapter contains the conclusions of this work and highlights the possible future work based on these results.

6.1 Conclusion

In this thesis, a full evaluation of null steering by perturbing the minimum number of elements along the axis of the array has been studied in detail for arrays with different number of elements and nulls. Null steering has been realized using genetic algorithm. The GA is very suitable for this type of study which can only be solved in an analytic form by guess work or trial & error method. The GA is very suitable because we have been able to incorporate the minimum no. of elements in to the fitness function and obtain a suitable solution for it. Fully and partially adaptive arrays of 8, 16 and 30 elements are studied for the cases of both uniform and Chebyshev arrays with single and multiple number of imposed nulls. In all the cases it is observed that

- The effect of reducing the number of controlled elements on the array parameters such as half-power beamwidth (HPBW), array directivity, sidelobe level (SLL) is negligible. The sidelobe variation (SLV), in case of partially adaptive arrays is slightly higher than fully adaptive case, which results due to the fact that the best array parameters are obtained, when all the elements are perturbed as it affords the greatest control over the array response.

- As we steer a null towards the main beam, the minimum number of controlled elements K and the sidelobe variation (SLV) increase. The effect on the remaining array parameters is negligible. The reason for this behavior is that as we are steering a null from a lower energy concentrated area to a higher energy concentrated area, it requires a higher order of degrees of freedom, hence larger number of controlled elements.

- When the sidelobe restriction is applied, the number of controlled elements K were increasing from the minimum possible value. This is due to increasing the number of constraints in this optimization problem.

- As the number of nulls increases, the minimum number of controlled elements K and the sidelobe variation (SLV) also increase, while the effect on remaining array parameters is negligible. The reason is that steering more nulls requires higher order of degrees of freedom, hence larger number of controlled elements.

- The elements have larger perturbation values when the null is imposed on sidelobe near to the main beam and the element perturbation values of partially adaptive array are slightly higher compared to the values of full element position perturbations. This is because the partially adaptive technique adds more value to the perturbed elements $\hat{\Delta}_n$, to compensate for those unperturbed elements. These large perturbation values of the partial method allow us to use less sensitive motors (lower precision) to apply the necessary mechanical movement to the elements.

- The minimum no. of controlled elements K, the range of element perturbations, the no. of generations and the CPU time required for null steering in chebyshev array is less compared to the uniform array, this is because of the low sidelobe level of chebyshev array. But the price we pay here for choosing array with low sidelobe level is a wider half-power beamwidth and low directivity.

- Application of a sidelobe constraint requires a high population size, more no. of generations and CPU time, which is due to addition of an extra constraint in this optimization problem.

- The 8-element partially adaptive array is suitable to steer only a single null. For the uniform array, the minimum no. of controlled elements vary between 2 and 4 depending on the location of the steered null. In order to be able to steer a single null over the whole sidelobe region, then 4 controlled elements out of 8 should be used. In order to be able to steer two nulls over most of the sidelobe region, then 6 controlled elements out of 8 should be used. For the chebyshev array, the minimum no. of controlled elements vary between 2 and 3 depending on the location of the steered null. In order to be able to steer a single null over the whole sidelobe region, then 3 controlled elements out of 8 should be used. In order to be able to steer two nulls over the whole sidelobe region, then 5 controlled elements out of 8 should be used.

- The 16-element uniform partially adaptive array is suitable to steer up to two nulls. For single null, the minimum no. of controlled elements vary between 3 and 7 depending on the location of the steered null. In order to be able to steer a single null over most of the sidelobe region, then 7 controlled elements out of 16 should be used. For two nulls, the minimum no. of controlled elements vary between 7 and 8 depending on the locations of the steered nulls. In order to be able to steer two nulls over most of the sidelobe region, then 8 controlled elements out of 16 should be used. In order to be able to steer three and four nulls over most of the sidelobe region, then 11 and 12 controlled elements out of 16 should be used respectively.

- The 16-element chebyshev partially adaptive array is suitable to steer up to four nulls. For single null, the minimum no. of controlled elements vary between 3 and 6 depending on the location of the steered null. In order to be able to steer a single null over the whole sidelobe region, then 6 controlled elements out of 16 should be used. For two nulls, the minimum no. of controlled elements vary between 6 and 8 depending on the locations of the steered nulls. In order to be able to steer two or three nulls over the whole sidelobe region, then 8 controlled elements out of 16 should be used. In order to be able to steer four nulls over the whole sidelobe region, then 9 controlled elements out of 16 should be used.

- The 30-element uniform partially adaptive array is suitable to steer up to three nulls. For single null, the minimum no. of controlled elements vary between 6 and 16 depending on the location of the steered null. In order to be able to steer a single null over most of the sidelobe region, then 16 controlled elements out of 30 should be used. For two nulls, the minimum no. of controlled elements vary between 10 and 18 depending on the locations of the steered nulls. In order to be able to steer two nulls over most of the sidelobe region, then 18 controlled elements out of 30 should be used. In order to be able to steer three, four, five and six nulls over most of the sidelobe region, then 21, 23, 24 and 26 controlled elements out of 30 should be used respectively.

- The 30-element chebyshev partially adaptive array is suitable to steer up to six nulls. For single null, the minimum no. of controlled elements vary between 6 and 10 depending on the location of the steered null. In order to be able to steer a single null over the whole sidelobe region, then 6 controlled elements out of 30 should be used. For two nulls, the minimum no. of controlled elements vary between 10 and 12 depending on the locations of the steered nulls. In order to be able to steer two nulls over the whole sidelobe region, then 12 controlled elements out of 30 should be used. In order to be able to steer three, four, five and six nulls over the whole sidelobe region, then 15, 17, 18 and 21 controlled elements out of 30 should be used respectively.

The results show that this technique of designing the partially adaptive arrays using genetic algorithms reduces the number of controlled elements to less than half. This leads to a significant reduction in the number of stepper motors used as compared to the fully adaptive implementation. Consequently, reduces the complexity, cost, the response time and at the same time maintains almost the same performance as that of fully adaptive arrays. In addition the genetic algorithm used steered the array nulls precisely to the required directions, achieved the prescribed null depth and at the same time reduced the sidelobe variation, which proves the effectiveness of this optimization technique to the design of partially adaptive arrays.

The resulting reduced complexity partially adaptive array have remarkably good sidelobe control over a wide range of interference conditions. For the case of arrays containing a large number of elements, this is a particularly useful approach to providing reduced cost, complexity and response time, while preserving the interference rejection capability of the array.

6.2 Future work

The study conducted in this work leads to the following future work

- The proposed genetic algorithm technique, with a slight variation in the fitness function can be applied to the design of partially adaptive endfire and scanned arrays.
- Research can also be directed towards the investigation of the performance of this genetic algorithm based partially adaptive technique on planar, circular and any arbitrary array.
- The experimental verification of null steering, by controlling minimum no. of array elements can be done by using the data in this thesis.
- Further experimental work can be directed towards a complete real time implementation of partially adaptive array using genetic algorithm technique.
- Investigation of the genetic algorithm optimization technique could be undertaken for null steering in partially adaptive arrays using phase only, amplitude only and complex weight methods.

Nomenclature

a_n	The current excitation of the n^{th} element
c_m	The beam coefficient of the m^{th} cancellation beam
C	An arbitrary constant
d_0	The unperturbed interelement spacing in wavelength
d_n	The unperturbed position of the n^{th} element
Δ_n	The n^{th} calculated element position perturbation
$\hat{\Delta}_n$	The n^{th} calculated partial element position perturbation
Δ_q	The q^{th} selected element position perturbation of an array with N elements
$F(u_s)$	The array factor in the main beam direction
$F(u_m)$	The array factor of the m's in the interfering signal directions
HPBW	Half power beam width
k	Wave number $\frac{2\pi}{\lambda}$
K	Number of controlled elements
λ	Operating wavelength of the array
SLL	Sidelobe level of the array pattern

SLV	Sidelobe variation of the array pattern
θ	Angle from broadside
θ_s	Main beam steering angle
θ_m	Angular location of the m^{th} jammer
WSR	With sidelobe restrictions
WOSR	Without sidelobe restrictions

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